On the Prediction Error for Singular Stationary Processes

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Let $X(t)$, $t \in \mathbb{Z} := \{0, \pm 1, \dots\}$, be a centered second-order stationary process with spectral density $f(\lambda)$, $\lambda \in [-\pi, \pi]$. Let $\sigma^2(f)$ denote the best linear prediction error of the random variable $X(0)$ by the entire infinite past: $\{X(t),\, t \le -1\}$, and let $\sigma_n^2(f)$ be the linear prediction error by a finite past of length n: $X(-n), \dots, X(-1)$.

From the prediction point of view it is natural to distinguish the class of processes for which we have *error-free prediction* by the entire infinite past, that is, $\sigma^2(f) = 0$. Such processes are called *singular* or *deterministic*. Processes for which $\sigma^2(f) > 0$ are called *regular* or *nondeterministic*.

Define the *relative prediction error* $\delta_n(f) := \sigma_n^2(f) - \sigma^2(f)$, and observe that $\delta_n(f) \ge 0$ and $\delta_n(f) \to 0$ as $n \to \infty$. But what about the speed of convergence of $\delta_n(f)$ to zero as $n \to \infty$? This speed depends on the regularity nature (regular or singular) of the observed process $X(t)$.

It turns out that for regular processes the asymptotic behavior of the prediction error is determined by the dependence structure of the observed process $X(t)$ and the differential properties of its spectral density f, while for singular processes it is determined by the geometric properties of the spectrum of $X(t)$ and singularities of its spectral density f.

The prediction problem we are interested in is *to describe the rate of decrease of* $\delta_n(f)$ *to zero as* $n \to \infty$, depending on the regularity nature of the observed process $X(t)$.

In this talk, we discuss the above problem for the less investigated case - singular processes. We provide extensions of Rosenblatt's and Davisson's results concerning asymptotic behavior and upper bounds for the finite prediction error $\sigma_n^2(f)$. Examples illustrate the stated results.

The talk is based on the joint works with Nikolay Babayan (Russian-Armenian University) and Murad Taqqu (Boston University).