Using state space models to understand rhythmic dynamics in the brain

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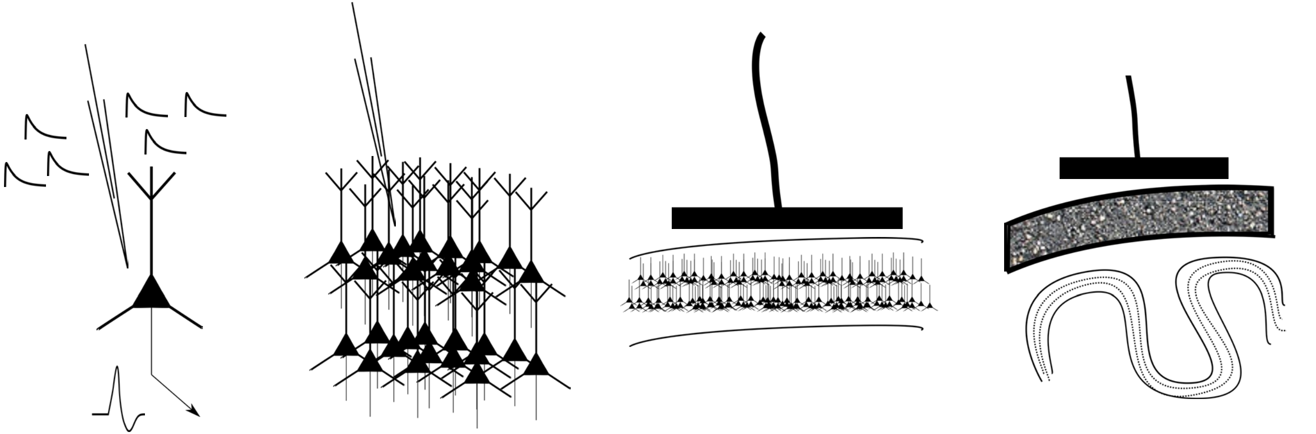
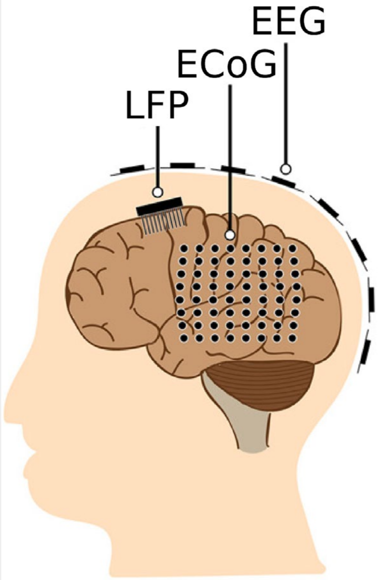
June 26, 2023

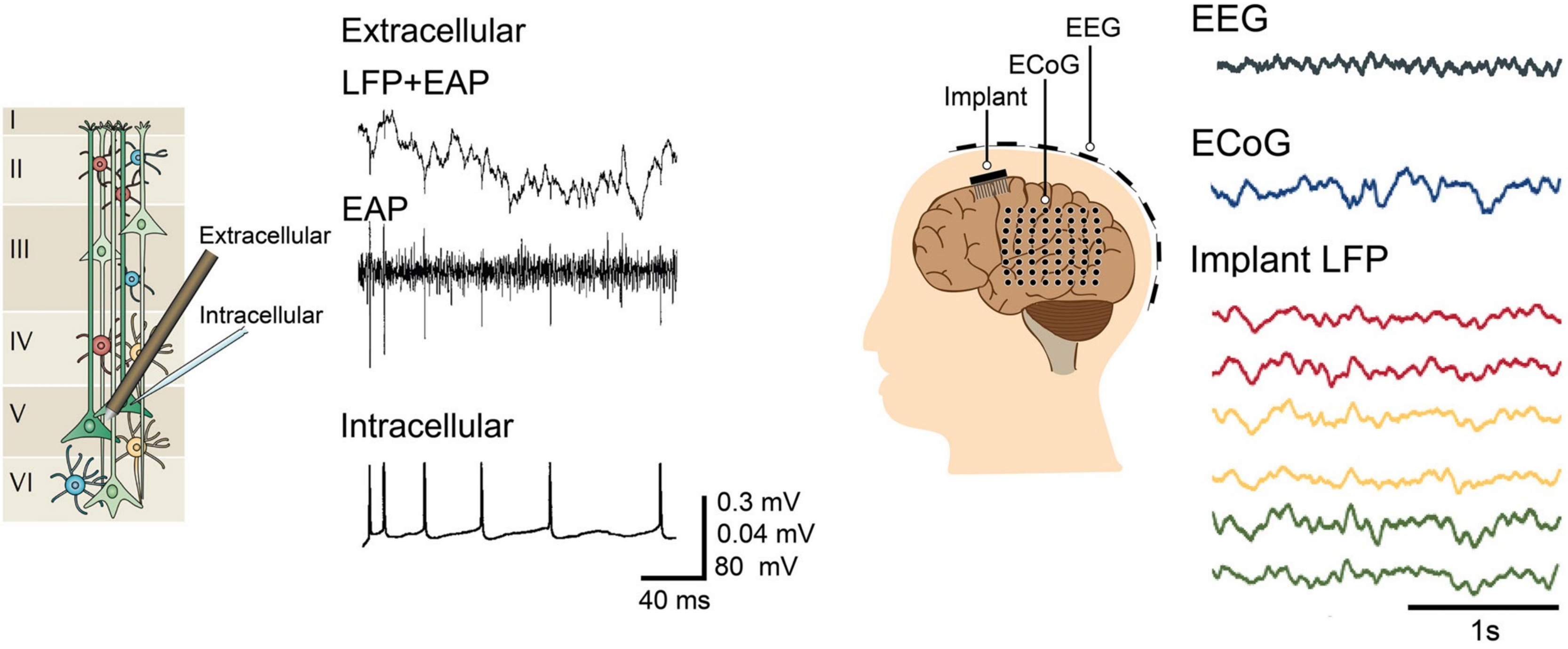
Electrophysiological voltage recordings





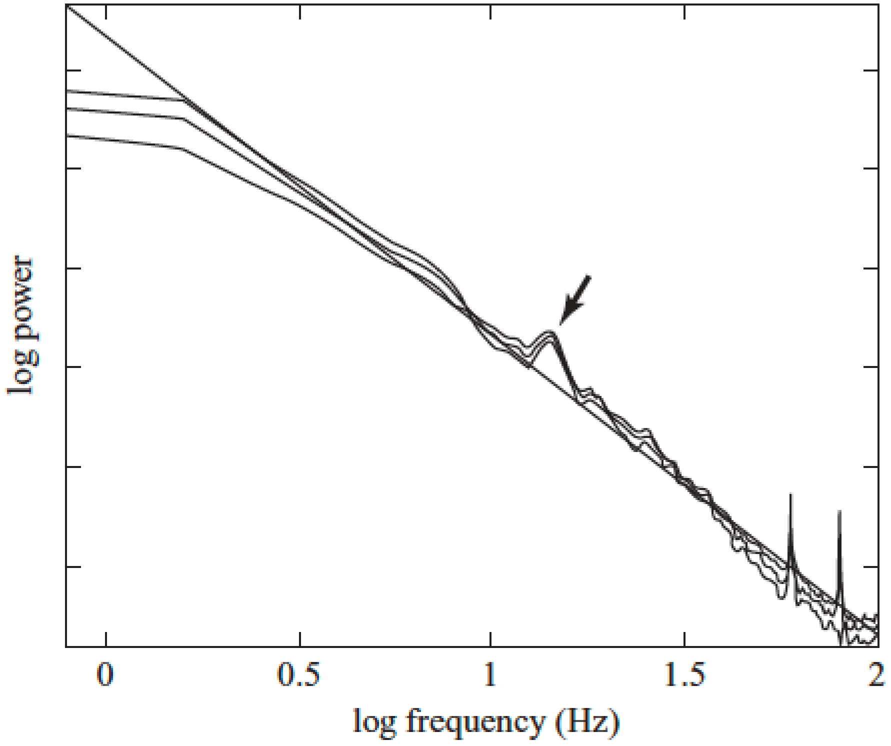
Single Unit	Local Field Potential	Electrocorticography	Electroencephalography
50 µm	(LFP)	(ECoG)	(EEG)
	Multiunit Activity	3mm	>2 cm
	300 µm		

Neural electrophysiology data (ephys) across scales



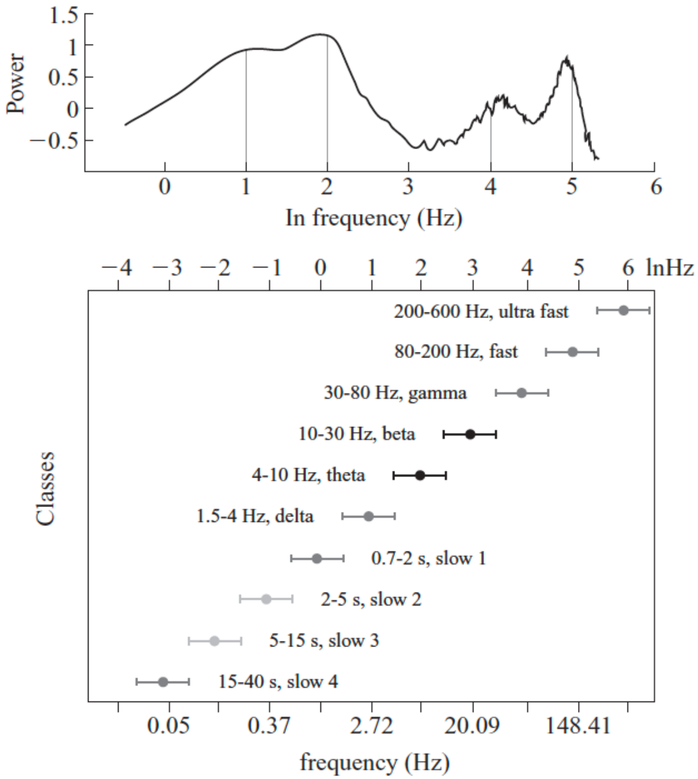
Obien et al (2015).

The ephys power spectrum has a "power law" or $1/f^{\alpha}$ shape (linear on log-log scale)



Buzsáki (2006) Rhythms of the Brain

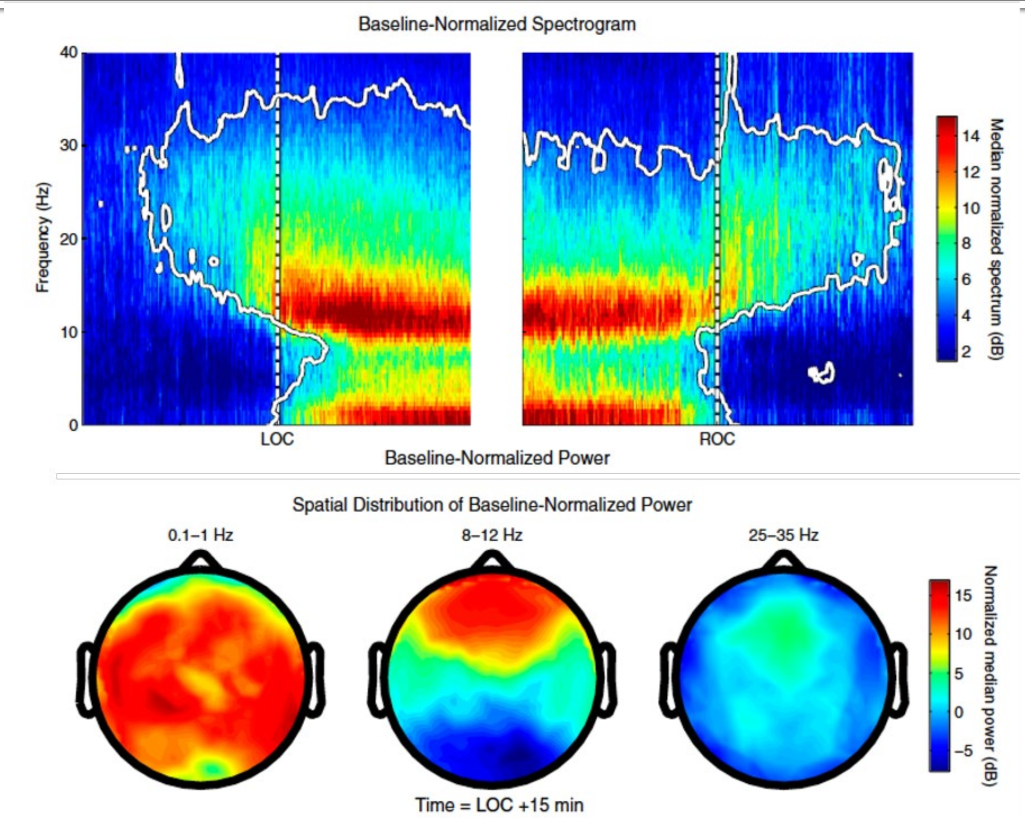
Rhythms are classically defined by narrow-band peaks in the power spectrum



(Here, they have removed the log slope)

Buzsáki (2006) Rhythms of the Brain

Propofol-induced unconsciousness has slow and alpha rhythms



Purdon et al (2013) PNAS

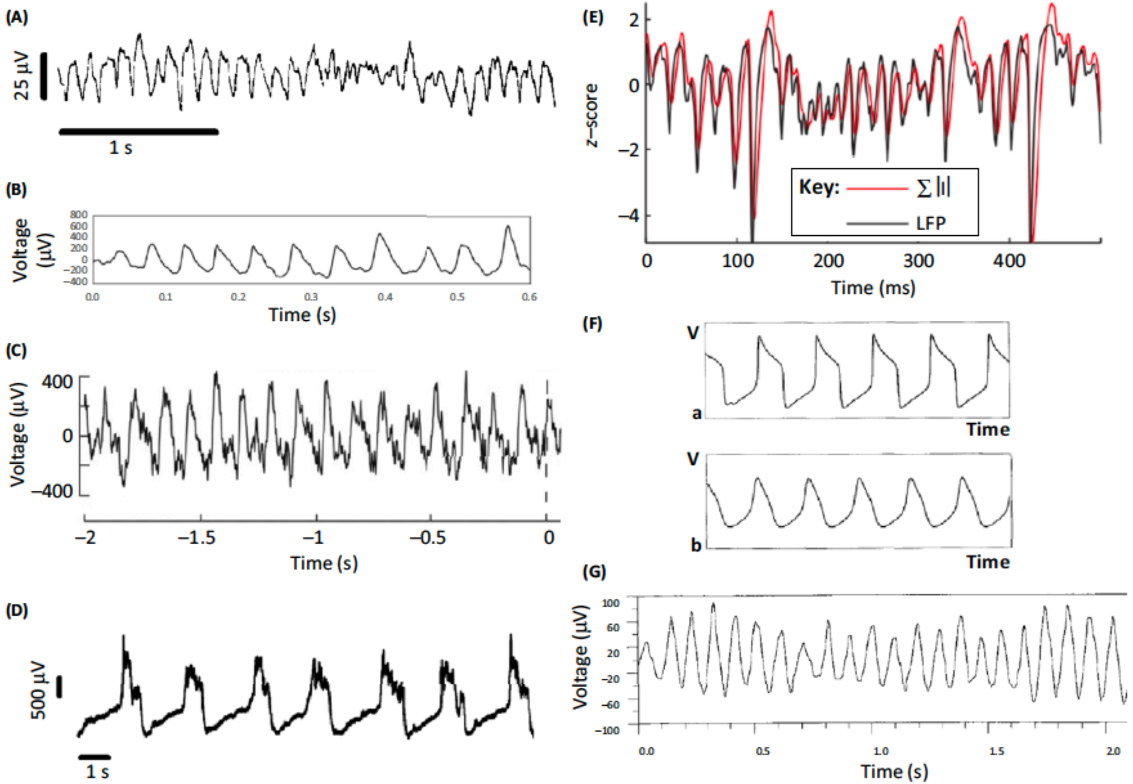
Rhythms are common (but not always oscillatory)

Human
EEG Mu

Human
ECoG
Beta

Rodent
LFP Theta

Rodent
LFP Slow



Simulated
PING

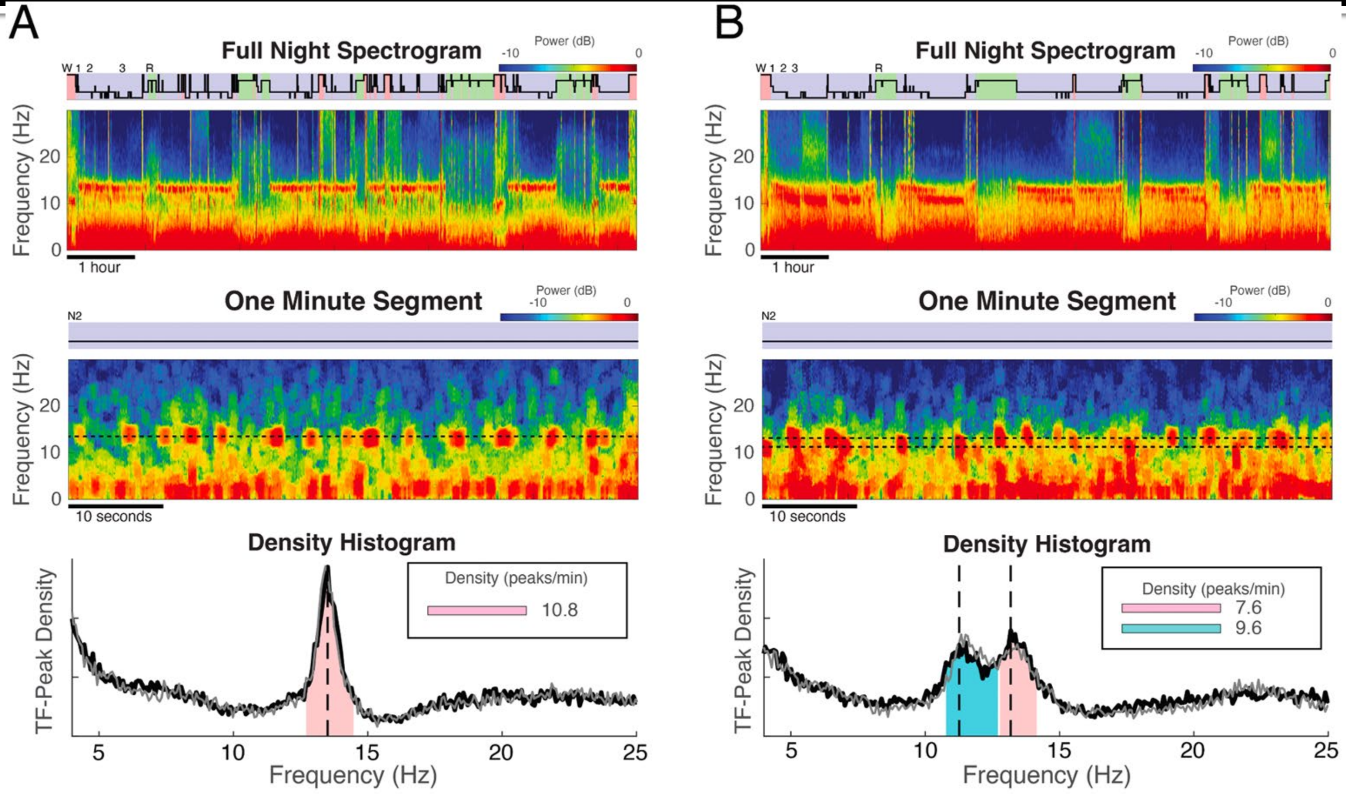
Simulated
Slow

Rodent
LFP alpha

Cole and Voytek (2017) Trends Cog Sci

Rhythms are common (but not always persistent)

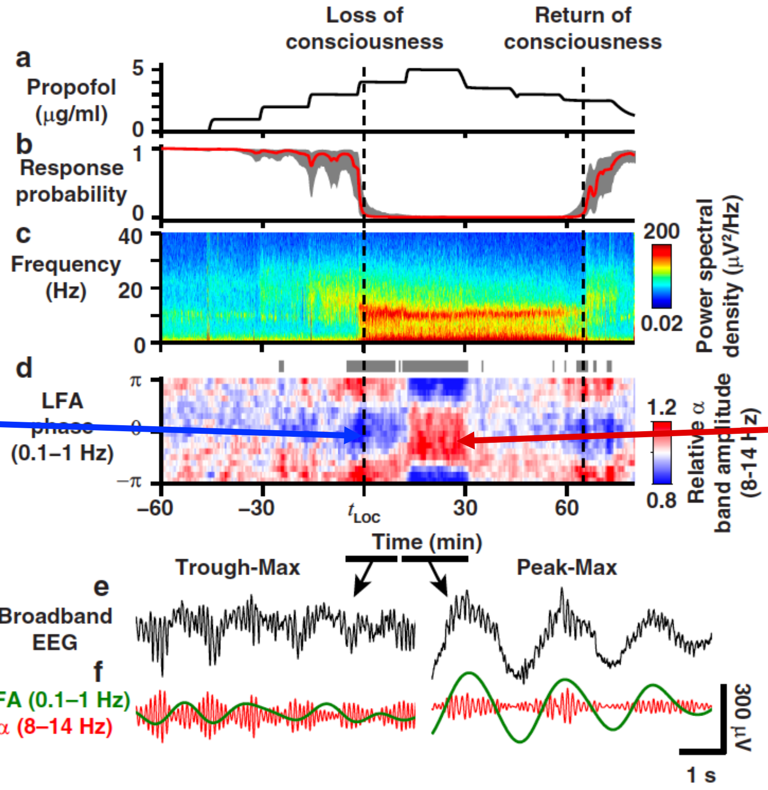
Sleep Spindles in two human participant
s



Stokes et al. (2023) Sleep

Rhythms can interact across frequencies

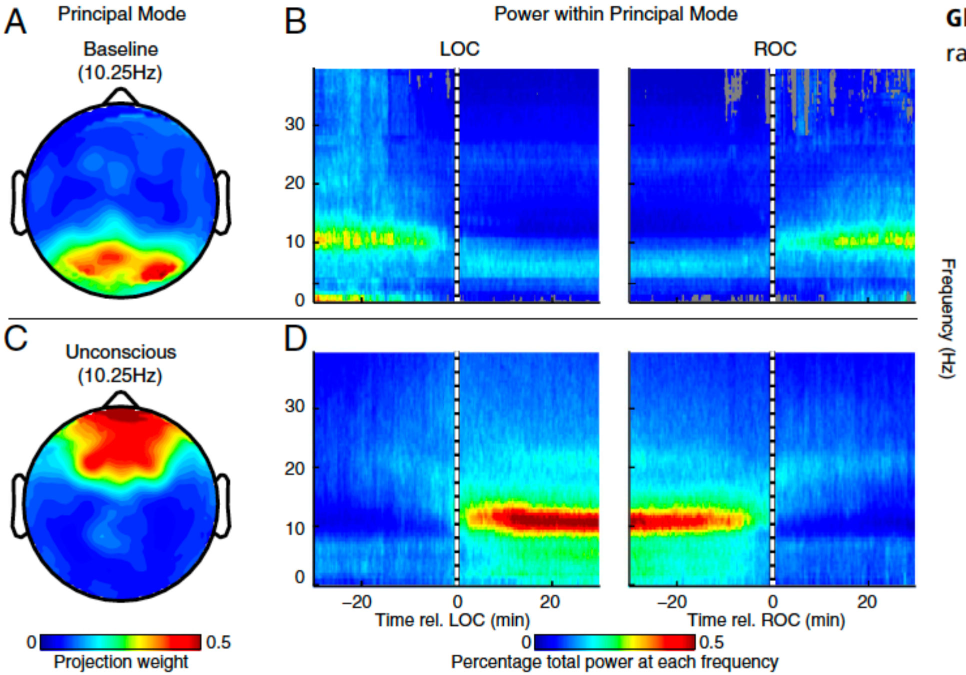
Troughmax can precede LOC



Peakmax only occurs when the subject is unarousable

Mukamel et al (2014) J Neurosci

Rhythms can interact across space



Global Coherence. Sorting the eigenvalues, $S_1^Y(f) \geq S_2^Y(f) \geq \ldots \geq S_N^Y(f)$, the ratio of the largest eigenvalue to the sum of eigenvalues is:

$$C_{\text{Global}}(f) = \frac{S_1^Y(f)}{\sum_{i=1}^{N} S_i^Y(f)}.$$

Amount of the total power that is captured by the first eigenvalue of the cross-spectral matrix

What is the best way to quantify all of these effects?

What is the best model for a rhythm?

Purdon et al (2013) PNAS

Agenda

- Defining rhythms
  - Sinusoids
  - Analytic Signal
  - State Space Oscillators
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Sinusoidal oscillations as phasors

Fourier Transform:

$$x_n = \frac{1}{N}\sum_{k=0}^{N-1} X_k e^{in\frac{2\pi k}{N}} = \frac{1}{N}\sum_{k=0}^{N-1} |X_k|\, e^{i\left(n\frac{2\pi k}{N}+\angle X_k\right)}$$

A discrete time signal
is an average of complex phasors
across frequencies,
each with starting phase $\angle X_k$
and amplitude $|X_k|$

$$|X_k| = \sqrt{X_{k,Re}^2 + X_{k,Im}^2}$$

$$\angle X_k = \text{atan2}\left(X_{k,Im}, X_{k,Re}\right)$$

Instantaneous Phase $\phi(t)$

But neural rhythms aren't sinusoidal!

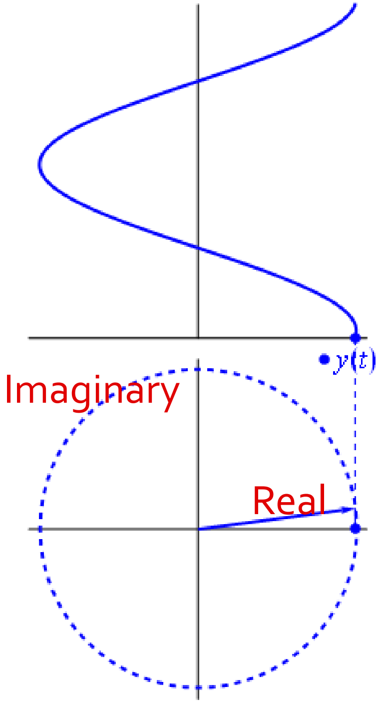
x_n

Signal = projection onto Real axis

$|X_k|\, e^{i\left(n\frac{2\pi k}{N}+\angle X_k\right)}$

Phasor

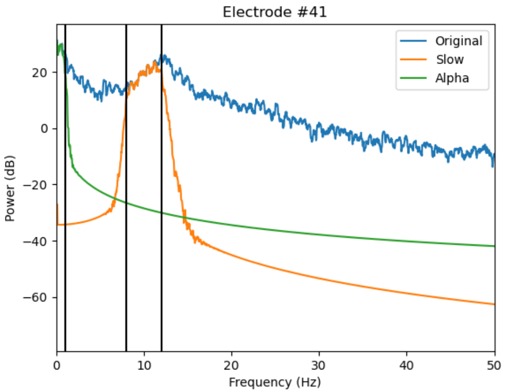
Amplitude = vector length
Phase = starting angle



https://upload.wikimedia.org/wikipedia/commons/8/89/Unfasor.gif

Standard Analysis: Bandpass filter, Analytic Signal

(1) Bandpass Filter



(2) Compute Analytic Signal

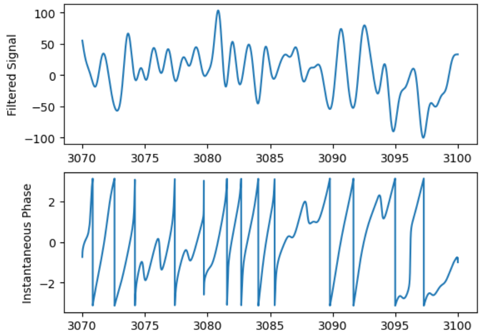
$$X_t^a = X_t + iH(X_t)$$

$$\approx a(t)e^{i(\omega_0 t + \theta_0)}$$

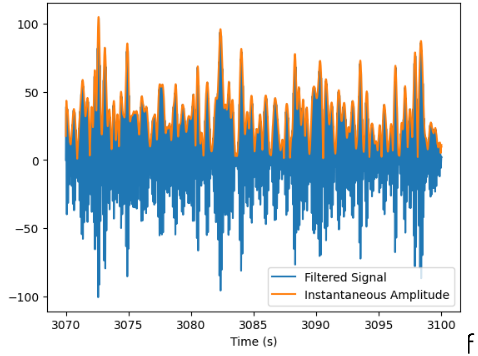
If X_t is *narrowband*

But bandpass filtering imposes arbitrary cutoffs that may leave out signal and include noise

(3a) Instantaneous Phase



(3b) Instantaneous Amplitude



Non-sinusoidal oscillations as noisy phasors

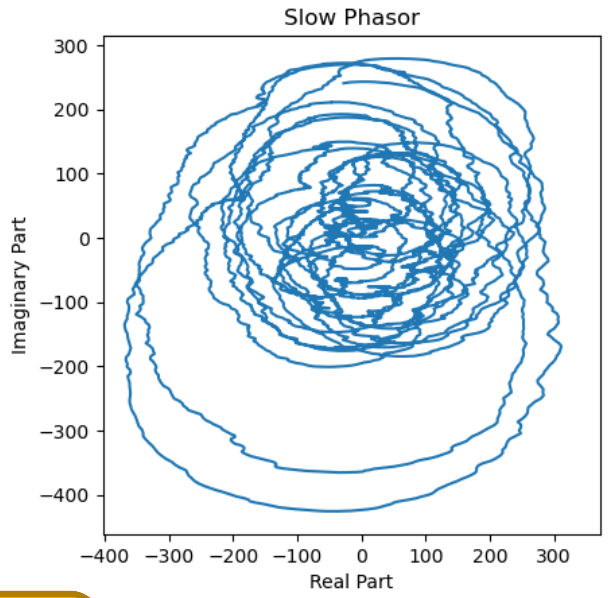
A rotating 2-D vector autoregressive process can be interpreted as a noisy phasor:

$$X[n] = \begin{bmatrix} X_{Re}[n] \\ X_{Im}[n] \end{bmatrix} = a \begin{bmatrix} \cos(2\pi f \Delta t) & -\sin(2\pi f \Delta t) \\ \sin(2\pi f \Delta t) & \cos(2\pi f \Delta t) \end{bmatrix} \begin{bmatrix} X_{Re}[n-1] \\ X_{Im}[n-1] \end{bmatrix} + \begin{bmatrix} v_{Re}[n] \\ v_{Im}[n] \end{bmatrix}$$

Decay (for stability)

Rotate by $2\pi f \Delta t$ at each time step

Gaussian Noise



This is the foundation of the State Space Oscillator (SSO) Model

The State Space Oscillator Model

Hidden state: K oscillators

$$\begin{bmatrix} x_{t+1,1}^{(k)} \\ x_{t+1,2}^{(k)} \end{bmatrix} = a \begin{bmatrix} \cos(2\pi f_k \Delta t) & -\sin(2\pi f_k \Delta t) \\ \sin(2\pi f_k \Delta t) & \cos(2\pi f_k \Delta t) \end{bmatrix} \begin{bmatrix} x_{t,1}^{(k)} \\ x_{t,2}^{(k)} \end{bmatrix} + \begin{bmatrix} v_{t,1}^{(k)} \\ v_{t,2}^{(k)} \end{bmatrix} \qquad k = 1 \ldots K$$

$$\begin{bmatrix} v_{t,1}^{(k)} \\ v_{t,2}^{(k)} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_k^2 & 0 \\ 0 & \sigma_k^2 \end{bmatrix} \right)$$

Observations: sum of real parts of the oscillators

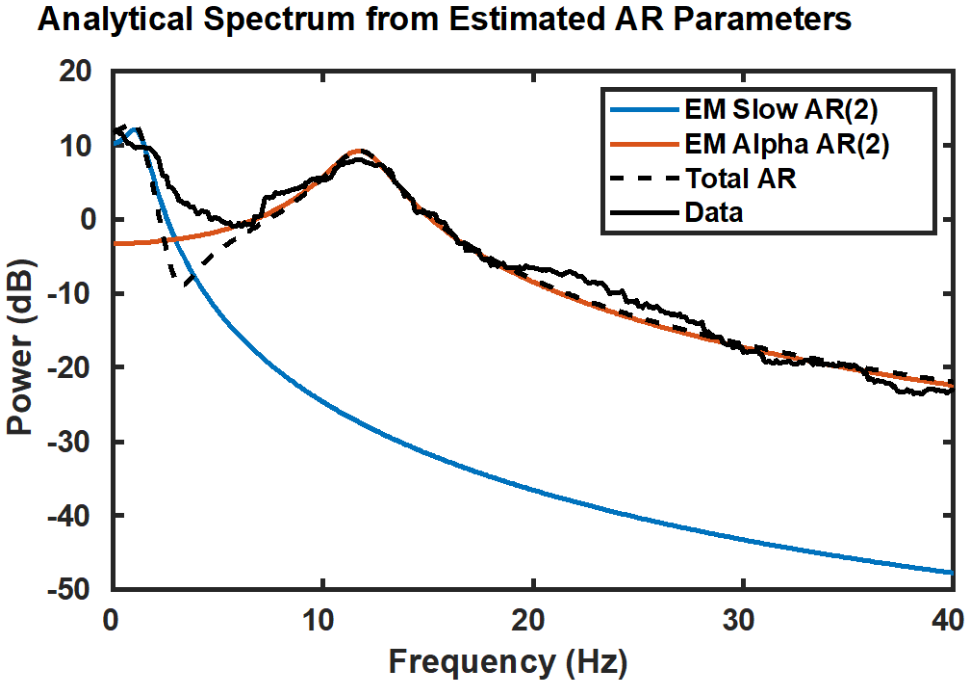
$$y_t = \sum_{k=1}^{K} x_{t,1}^{(k)} + w_t \qquad w_t \sim N(0, \tau^2)$$

Inst. Amplitude $A_t^{(k)} = \sqrt{x_{t,1}^{(k)2} + x_{t,2}^{(k)2}}$

Inst. Phase $\phi_t^{(k)} = \operatorname{atan2}\left(x_{t,2}^{(k)}, x_{t,1}^{(k)} \right)$

Matsuda and Komaki (2017). Neural Computation

The model process has a parametric power spectrum, where each rhythm is broadband



Beck, Stephen, Purdon (2018) IEEE EMBC

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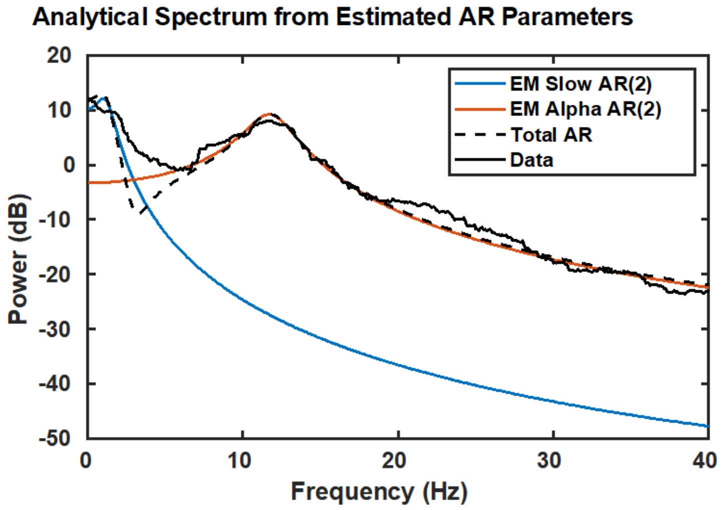
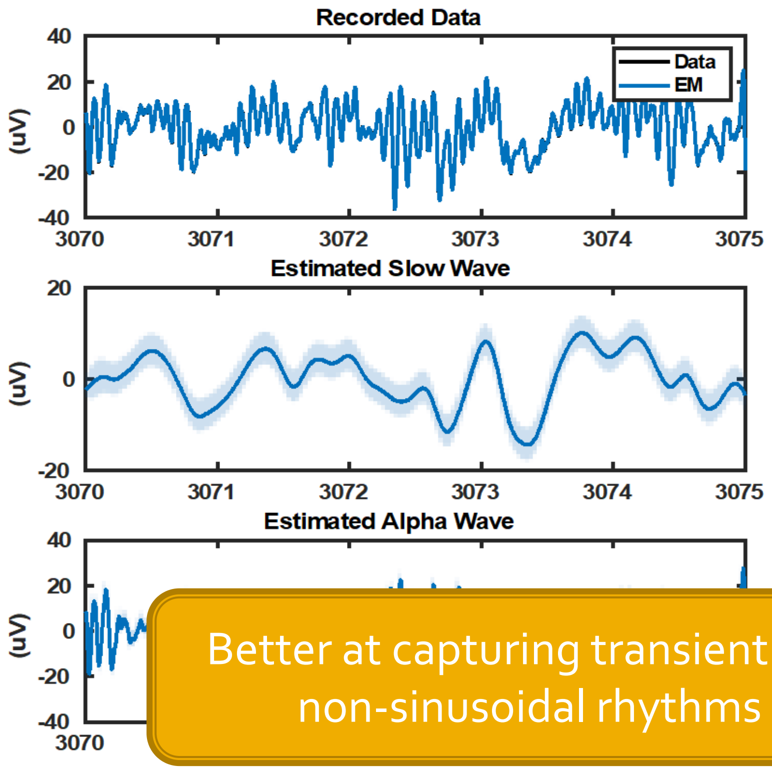
Can use classic state space techniques to estimate the hidden states and parameters (EM, filters/smoothers)





Better at capturing transient and non-sinusoidal rhythms

Beck, Stephen, Purdon (2018) IEEE EMBC

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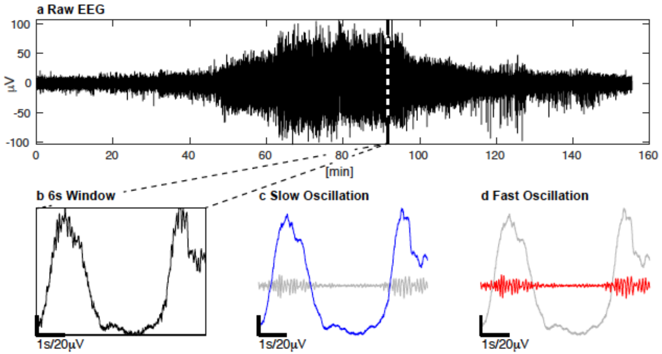
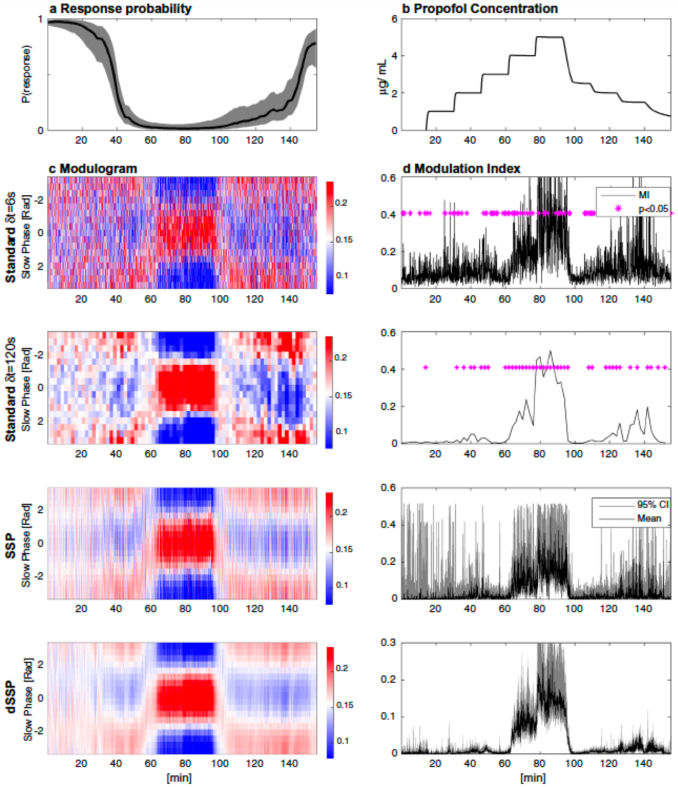
Using the phase and amplitudes from the fitted model, we can estimate phase amplitude coupling



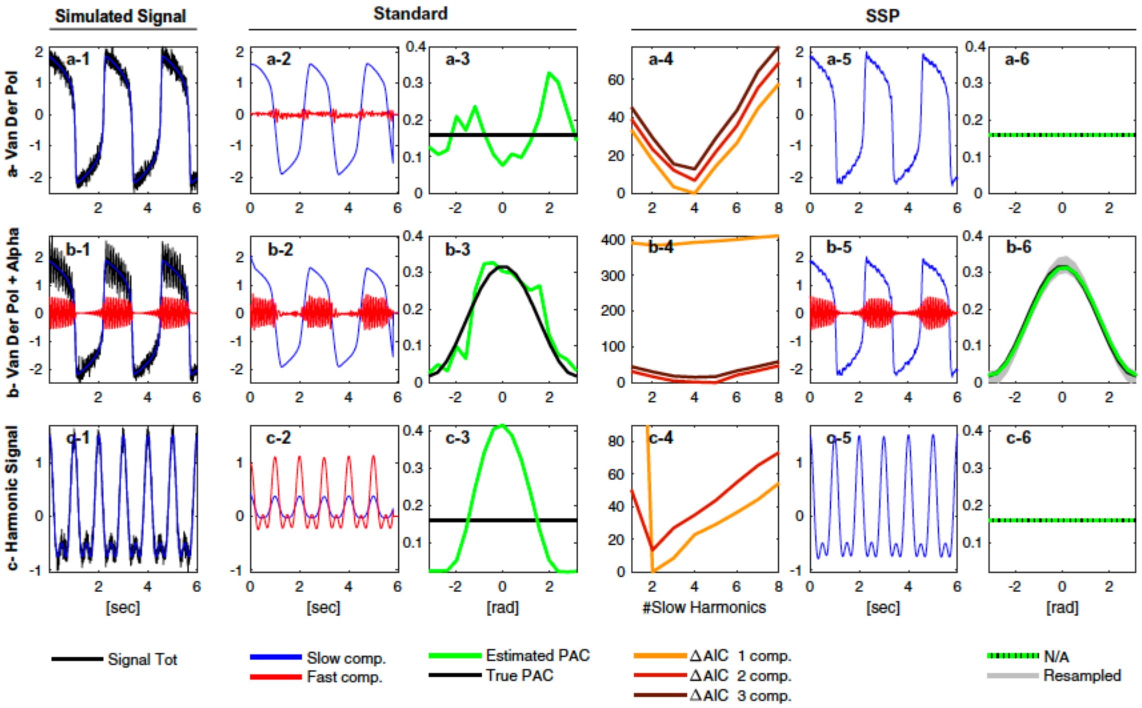
Soulat, Stephen, Beck, Purdon (2022) Scientific Reports

The resulting models are much more powerful than standard techniques



Soulat, Stephen, Beck, Purdon (2022) Scientific Reports

The models are more robust to nonsinusoidal and harmonic signals



Soulat, Stephen, Beck, Purdon (2022) Scientific Reports

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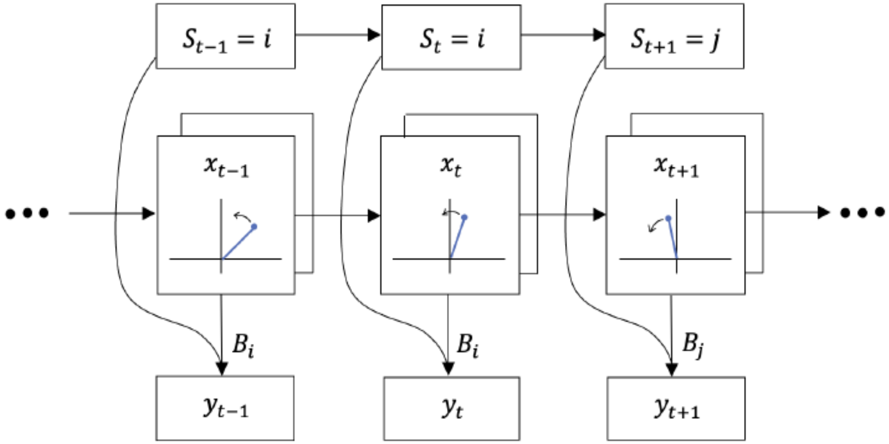
Using SSOs for functional connectivity estimation with switching states

Common Oscillator Model



Multivariate observations

$$P(S_t = j | S_{t-1} = i) = Z_{ij}$$
$$(x_t | x_{t-1}, S_t = j) \sim N(A_j x_{t-1}, \Sigma_j)$$
$$(y_t | x_t, S_t = j) \sim N(B_j x_t, R)$$

Common Oscillator Model

Correlated Noise Model

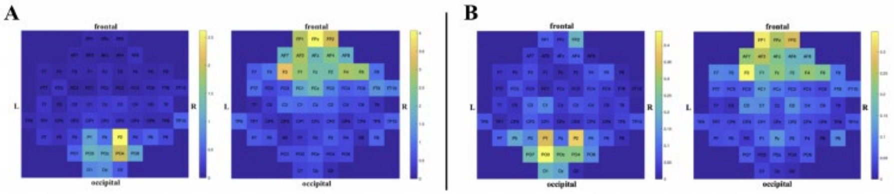
Directed Influence Model

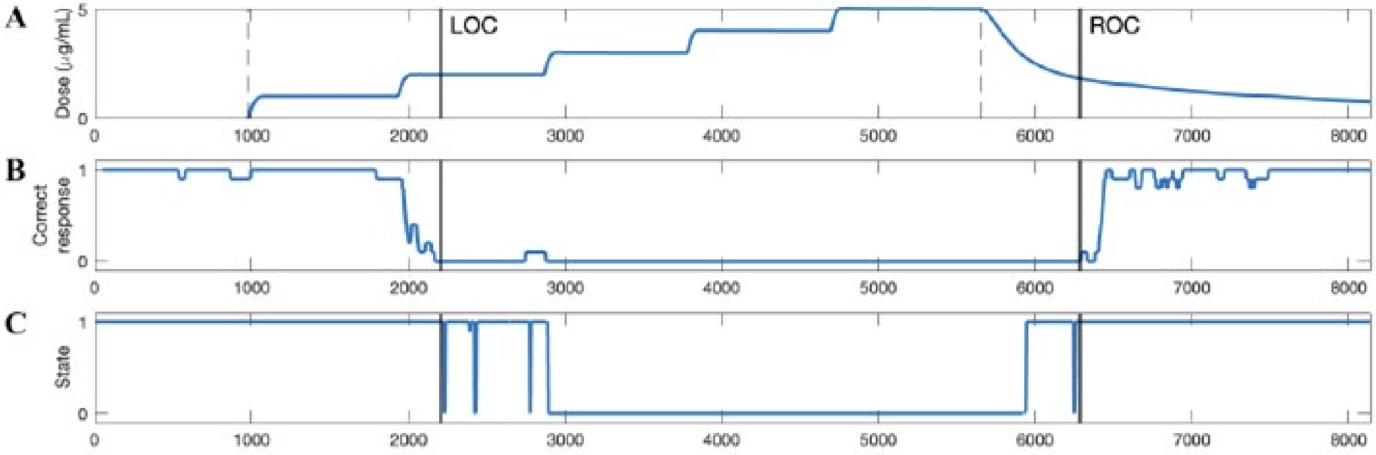
Hsin, Eden, Stephen (2022) IEEE SSC

The Common Oscillator Model can capture switching global coherence modes at fine time resolution

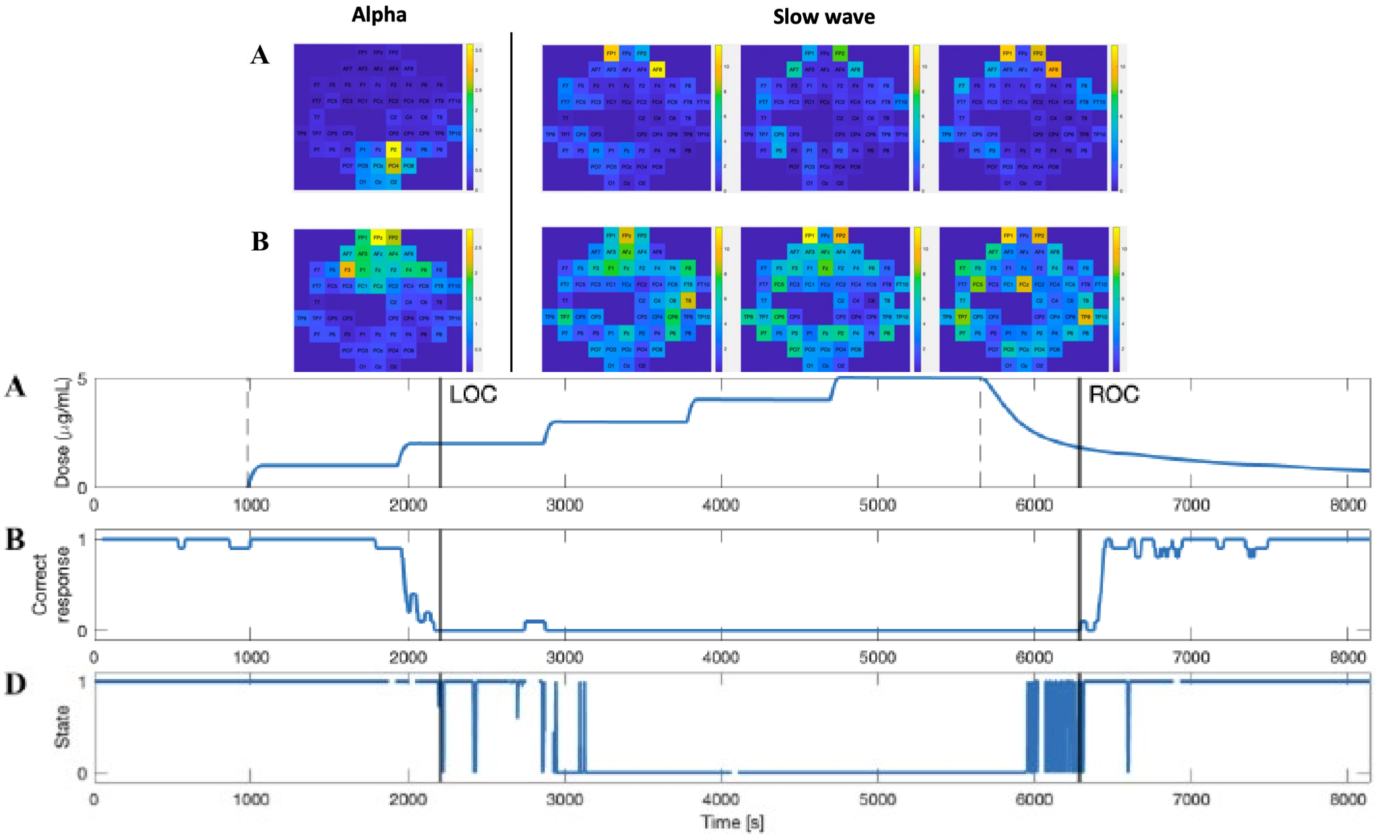
Common Oscillator
Alpha rhythm B matrices

Global Coherence
Magnitude at alpha





The Common Oscillator model can learn switches from multiple overlapping rhythmic networks

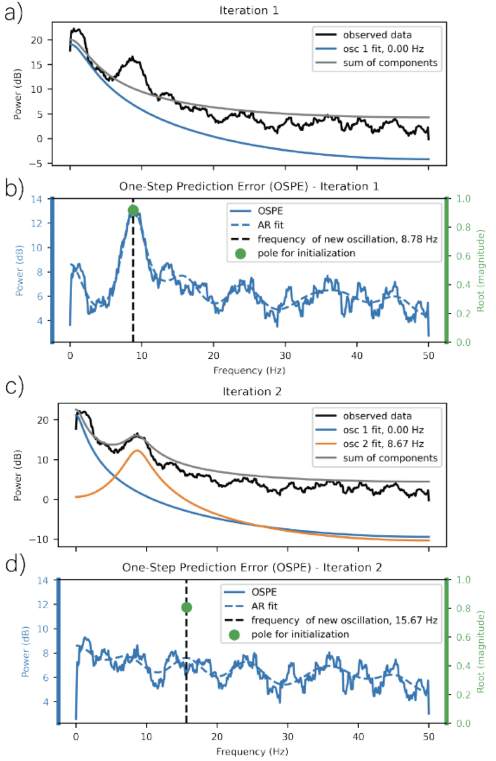


Conclusions

- State space oscillators are a powerful, flexible framework for capturing neural oscillations
  - Decomposing signals in the time domain
  - Phase amplitude coupling
  - Functional networks

- Future directions:
  - Other network models and applications
  - Other observation models

You can build the model with an iterative search



Beck, He, Gutierrez, Purdon (2022) Neuroscience