6-2-2014 Fabrizio Andreatta Glennfest 15:45 pm

Towards a crystalline overconvergent
Eichler–Shimura isomorphism

(30 min Hodge-Tate analogue of Eichler-Shimura.
15 min new work jt with Adrian Iovita)

K, CDVF of char 0.
with perfect res. field of char $p>2$.
$v(p)=1$
$\underline{\zeta_p \in K}_{n}$. (to be explicit)

$N \in \mathbb{Z}_{\geq 4}$, prime to p.

(1) Integeral p-adic Hodge theory
and overconvergent modular forms.
(jt. with Iovita, Pilloni, and Stevens)

$$\begin{array}{c} E \\ \downarrow \\ \operatorname{Spec} R \\ \downarrow \\ \operatorname{Spec} \mathcal{O}_K \end{array}$$

E, abelian scheme of dim g

R, p-adically ~~cpt~~ étale / semistable

$\mathcal{O}_K\{X_1, \dots, X_d, X, Y\} / (XY-\pi)$

$\pi \in \mathcal{O}_K$ uniformizer

Assume E is "almost ordinary".

$F = \mathrm{Frob}^* : H^1(E, \mathcal{O}_E/p\mathcal{O}_E)^{(p)} \to H^1(E, \mathcal{O}_E/p\mathcal{O}_E)$

$\det(F) \in R/pR \ni p^w$ class of an elt $p^w \in \mathcal{O}_K$ s.t. $v(p^w) = w < \frac{1}{p}$.

$w = 0 \iff E$ has ordinary reduction.

canonical subgp. + **Thm** $\exists\, C \subseteq E[p]$ canonical subgp. lifting to $\mathcal{O}_K$ the kernel of Frobenius on E.

<u>Galois side</u>.

Tate module of E.,

Namely, $R \subseteq \bar{R}$ "max'l ext'n unram. away from p"

$$T_p(E) = \varprojlim_n E[p^n](\bar{R}[\tfrac{1}{p}])$$

$$= \varprojlim_n E[p^n](\bar{R}).$$

$$\boxed{\mathcal{G}_R = \mathrm{Gal}(\bar{R}/R) \text{ acts on } T_pE \cong \mathbb{Z}_p^{2g}.}$$

dual. $\quad \otimes_{\mathbb{Z}_p} \hat{\bar{R}}$ $\leftarrow$ p-adic completion of $\bar{R}$.

$$\mathrm{dlog} : T_p E^{\vee} \longrightarrow \omega_{E/R} \otimes_R \hat{\bar{R}}$$

mod p^m

$$E^{\vee}[p^m](\bar{R}) \simeq \mathrm{Hom}_{\bar{R}}(E[p^m] \otimes \bar{R}, \mu_{p^m} \otimes \bar{R})$$

$$\ni \gamma \mapsto \gamma^*\left(\frac{dT}{T}\right) \in \omega_{E/R} \otimes \bar{R}/p^n\bar{R}.$$

<u>Note</u>. dlog is surjective

$\Longleftrightarrow$ E has ordinary reduction.

<u>Thm</u>. Under almost ordinarity,

(1) Img of dlog is a projective rank g $\hat{\bar{R}}$-module.

(2) If $C(R) = C(\bar{R})$, then

$$\Omega_{E/R} = \mathrm{Im}(\mathrm{dlog})^{G_R}$$ is a projective rank g R-module.

$$\Omega_{E/R} \subset \omega_{E/R}$$

$=$ $\Longleftrightarrow$ ordinary reduction.

s.t. $\text{dlog} \pmod p$

$$\begin{tikzcd}
\check{E}[p](\overline{R}) \otimes \overline{R}/p^{\square} \arrow[r] \arrow[d] & \Omega_{E/R} \otimes_R \overline{R}/p^{1-\frac{w}{p-1}}\overline{R} \\
\mathbb{F}_p^g \simeq C^\vee(\overline{R}) \arrow[ur] &
\end{tikzcd}$$

and a basis of $C^\vee(\overline{R})$ defines

an $R/p^{1-\frac{w}{p-1}}$-basis

of $\Omega_{E/R} / p^{1-\frac{w}{p-1}} \cdot \Omega_{E/R}$.

<u>Application</u> to construct sheaves of p-adic (Siegel/Hilbert) elliptic ~~an~~ overconvergent forms

<u>Fix</u> $\chi : \mathbb{Z}_p^\times \to K^\times$ analytic

$B_v \quad v \leq w$

affinoid in wt sp.

$\rightsquigarrow \Omega^\chi(w)$.

~~$I\!E$'s~~ ~~2~~

It's ~~values~~ elements are ~~or~~ Rules: (E/R, level N str, ψ_N, Ω (generator of $\Omega_{E/R}$), γ generator of $C^\vee(R)$)..

$$\text{s.t. } \Omega \equiv \gamma \mod p^{1-\frac{w}{p-1}}.$$

$$: (E/R, \psi_N, \Omega, \gamma) \mapsto f(E/R, \psi_N, \Omega, \gamma) \in R \hat{\otimes} B_u$$

s.t. $\alpha \in (1 + p^{1-\frac{w}{p-1}} \cdot R) \cdot \mathbb{Z}_p^\times$

$$f(-, -, \alpha \cdot \Omega, \bar{\alpha} \cdot \gamma)$$

$$\longmapsto \chi(\alpha)^{-1} \cdot f(E/R, \psi_N, \Omega, \gamma)$$

Eichler - Shimura map for modular symbols.

$g = 1$. $T_0 \subseteq T = Ta_p(E) = Ta_p(E^\vee)$ inverse image of $T \to C^\vee(\bar{R}) \supseteq C(\bar{R}) \setminus \{0\}$.

wide open ~~of~~ wt sp.

Fix a level str.

$$\Gamma = \Gamma_0(p) \cap \Gamma_1(N)$$

$X(N, p)$ modular curve.

$\mathcal{D}_\mathcal{U}$ = $B_\mathcal{U}$-valued analytic distributions on T_0.
homogeneous of deg $(\chi_\mathcal{U})$
for the action of $\mathbb{Z}_p^\times$ on T_0.
$\chi_\mathcal{U} : \mathbb{Z}_p^\times \to B_\mathcal{U}$ universal char.

<u>Thm</u> (A-I-S)

The map dlog defines for $\begin{matrix} E \\ \downarrow \\ \mathrm{Spec} R \end{matrix}$

as before.
a map $\mathrm{dlog}^{\chi_\mathcal{U}} : \mathcal{D}_\mathcal{U} \to \Omega_{E/R}^{(\mathcal{U})} \hat{\otimes} R$.

Induces a $\mathrm{Gal}(\bar{K}/K)$-equivariant
and Hecke equivariant map

$$ES : H^1(\Gamma, \mathcal{D}_\mathcal{U})(1)^{(\leq h)}_{\hat{\otimes} \mathbb{C}_p} \to \varinjlim_{w(\to 0)} H^0(X(p,N)(w), \Omega^{\chi_\mathcal{U}+2})^{(\leq h)} \hat{\otimes} \mathbb{C}_p.$$

<u>We are after</u>:

A_{cris}, also possible.

$$ES_{crys}: \left(H^q(\Gamma, \mathcal{O}_\mathcal{U})^{(h)}(1) \otimes B_{cris}\right)^{G_K} \longrightarrow ?$$

Fontaine's ring.