Explicit computation of $p$-adic variation of homology (distribution modules).

compute.

(I) Congruence between Hecke eigenspaces in $H_g (\Gamma, V_{\lambda})$ and $H_g (\Gamma, V_{\lambda'})$

in the rep's with highest at $\lambda$

mod $p$-power if $\lambda \equiv \lambda' \pmod{(p-1) \cdot p^\alpha}$.

Today's Focus

$\Gamma \subset \text{SL}_3(\mathbb{Z})$

$\lambda = (a, b, c) \in \mathbb{Z}^3$.

$a \geq b \geq c$.

To interpolate the coefficients, replace $V_{\lambda}$ (f.d. sp.)

by $D_{\lambda}$ ($\infty$-dim sp)

up modules of $p$-adic distributions.
\( D_{\lambda} \)

- They are Verma modules so can be defined like
- for any \( p \)-adic \( \lambda \).

- They are \( p \)-adic Banach spaces
- They vary \( p \)-adically and trivially in \( \lambda \).

- Have a comparison between

\[
H^\lambda (\Gamma, D_{\lambda}) \stackrel{\leq}{\to} H^\lambda (\Gamma, V_{\lambda}) \]

(Induced by a homomorphism \( D_{\lambda} \to V_{\lambda} \).)

if \( \lambda \) is the dominant integral weight.

If \( \lambda \) is a positive number

\( m(\lambda) \)
\[ \frac{GL_3}{U_1} \]

\[ B, \text{ upper triangular.} \quad \varepsilon B = B^{opp}. \]

\[ N \cdot T \]

\[ I, \text{ Iwahori subgroup} \subseteq GL_3(\mathbb{Z}_p). \]

\[ \Gamma' = \Gamma_0(M) \cap I. \]

\[ X = "\text{big cell}" \]

\[ = \text{image of } I \text{ in } T \]

\[ \Upsilon = N^{opp} \cdot GL_3(\mathbb{Q}_p) \backslash GL_3(\mathbb{Q}_p) \]

\[ \lambda : T(\mathbb{Z}_p) \rightarrow \mathbb{C}_p^\times \text{ conti. char.} \]

Torus action → \[ A_\lambda = \exists f : X \rightarrow \mathbb{C}_p : f(\pm x) = \lambda(\pm). \text{ Ax: } f \text{ is rigid analytic.} \]
\[
A \begin{pmatrix}
1 & x & y \\
0 & 1 & z \\
0 & 0 & 1
\end{pmatrix} = \sum C_{ijk} \cdot x^i \cdot y^j \cdot z^k
\]

\[C_{ijk} \rightarrow 0.\]

(assuming \(x\) is analytic)

\[D_\lambda = \text{Hom}_{\text{cts}}(\mathbb{A}_\lambda, \mathbb{C}_p) \otimes \mathbb{Q}_p \mathcal{G}_\lambda \Gamma^u\]

\[\sum b_{rst} \cdot \delta_r^2\]

We need

1) A way to compute \(H_3(\Gamma, A)\) for any module \(A\).

2) A way to compute Hecke operators.
Joint with Daniel Pollack. \( h < m(\lambda) \) critical slope.

3) A way to lift \( H_3(\Gamma, V_{\lambda_0}) \leq h \)
    to \( H_3(\Gamma, D_{\lambda_0}) \leq h \).

4) Deform from \( H_3(\Gamma, D_{\lambda_0}) \) to nearby \( H_3(\Gamma, D_{\lambda_1}) \), e.g.,
    eigenvalues projection \( \Rightarrow \) \\
    can get the tangent vector \( \Rightarrow \) e.g.,
    (hopefully e.g.,)

    idea. \( \frac{F|U^n}{\alpha^n} \rightarrow \text{eigenclass.} \)