6-2-2014 Eugen Hellmann Glennfest 14:30 pm.

Eigenvarieties, spaces of trianguline rep'ns, and patching.

jt work w/ C. Breuil
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Setup: · $F/\mathbb{Q}$ CM field.

$F^+ \subsetneq F$ tot. real.

· for simplicity, assume $\exists$ unique place of F^+ dividing p

$$L = F^+_{v_0}$$

G/F^+ · unitary gp. in n-variables split over F

· $G(F^+ \otimes_{\mathbb{Q}} \mathbb{R})$ cpt.

· quasi-split at all finite places

· split at v_0,

fix $G(F^+_{v_0}) \simeq GL_n(L)$

$$\begin{array}{c} GL_n(L) \\ \cup \\ B \\ \cup \\ T. \end{array}$$

fix a tame level $U^p \subseteq G(\mathbb{A}_{F^+}^{\infty, v_0})$
open cpt.

$\mathcal{H}$ = Hecke alg. at places where U^p is max'l cpt.

Eigenvariety for (G, U^p)
rigid space Y + $\psi : \mathcal{H} \to \Gamma(Y, \mathcal{O}_Y)$
$\delta : T \to \Gamma(Y, \mathcal{O}_Y^\times)$ conti. char.

Roughly: $y \in Y$ rigid pt.

$\rightsquigarrow \pi = (\pi_f)^{U^p} \otimes \pi_p$ in $\hat{S}(U_1^p, E)$

finite extn $/\mathbb{Q}_p$.

$G(F^+_{v_0}) = GL_n(L)$

$\mathcal{H} \circlearrowright \varprojlim_n \varinjlim_{U_p} \left\{ f : G(F^+) \backslash G(\mathbb{A}_{F^+}^\infty) / U_p \cdot U^p \to \mathcal{O}_E / \varpi^n \right\}$

$[\frac{1}{p}]$

$\rightsquigarrow$ p-adic completion of automorphic forms

$$\pi = (\pi_f)^{U^p} \otimes \pi_p$$

$\circlearrowleft \mathcal{H}$ $\qquad$ $\circlearrowleft GL_n(L)$

s.t. ~~$\psi \otimes \kappa(y)$~~

$\psi \otimes \kappa(y)$ Hecke char $\simeq (\pi_f)^{U^p}$.

and $\qquad$ $N_0 \subseteq N$ maxi'l cpt.

$$0 \neq \mathrm{Hom}_T\left(\delta \otimes \kappa(y), (\pi_p^{an})^{N_0}\right).$$

where N_0 = ~~maxi'l~~ open cpt

$\subseteq$ unipotent radical of B.

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y, <u>classical</u> if π comes from a classical automorphic form.

$\iff \pi_p^{alg} \neq 0$.

Then
- classical pts are dense.
- for y classical, $\exists$ Galois rep'n attached to π $\quad$ ρ_π or ρ_y
- $\exists$ family of $\mathrm{Gal}(\overline{\mathbb{Q}}/F)$-rep'ns on Y. interpolating automorphic Galois rep'ns at classical points.

$\mathcal{O}_E/\varpi\mathcal{O}_E$

Fix $\bar{\rho} : Gal(\bar{\mathbb{Q}}/F) \to GL_n(\mathbb{F})$

abs. irred.

$\Upsilon_{\bar{\rho}} \subseteq \Upsilon$ open closed. subsp.

conn. cpnt.

when the family of Galois rep'ns reduces to $\bar{\rho}$ mod p.

Let $\mathfrak{X}_{\bar{\rho}}$ be the rigid generic fiber of the univ. deform. ring of $\bar{\rho}$.

Get $\Upsilon_{\bar{\rho}} \hookrightarrow \mathfrak{X}_{\bar{\rho}} \times \mathcal{J}^n$ closed immersion.

space of conti. char. of $(L^\times)^n = T$.

$y \in \Upsilon \rightsquigarrow (\pi, \psi, \delta)$

$\pi_p \overset{??}{\longleftrightarrow} \rho_y|_{Gal(\bar{\mathbb{Q}}_p/L)}$

under hypothetical p-adic local Langlands correspondence ??

<u>Question</u> What should $0 \neq \mathrm{Hom}_T(\delta, (\pi_p^{an})^{N_0})$ mean for the assoc. local Galois rep'n?

<u>Should mean</u>:

$r_y := \rho_y |_{Gal(\overline{\mathbb{Q}}_p/L)}$ is <u>trianguline</u>

$D^{\dagger}_{rig}(r_y)$ assoc. (φ, Γ)-module over the Robba ring.

is an ext'n of <u>rank 1 objects</u>.

$\updownarrow$

characters. $\delta = (\delta_1, \cdots, \delta_n)$

$\delta_i : L^\times \to k(y)^\times$ defines a rank 1 (φ, Γ)-module over the Robba ring.

<u>Space of trianguline rep'ns</u>.

Fix $\overline{r} = \overline{\rho}|_{Gal(\overline{\mathbb{Q}}_p/L)}$. (abs. irred)

$\mathfrak{X}_{\overline{\rho}}$ = generic fiber of univ. deform. ring.

$$X_{tri}(\bar r) = \text{Zariski-closure of } (r, \delta_1, \dots, \delta_n)$$

$$\cap$$

$$\mathcal{X}_{\bar r} \times \mathcal{T}^n$$

s.t. r trianguline with graded pieces given by $\delta_1, \dots, \delta_n$

(δ is generic enough)

Relying on Kedlaya–Pottharst–Xiao, all rep'ns occurring in $X_{tri}(\bar r)$

- are trianguline generically with parameter $\delta_1, \dots, \delta_n$.
- equi-dim'l of dim $1 + [L:\mathbb{Q}_p] \cdot \dfrac{n \cdot (n+1)}{2}$.

$\bar\rho$ global

$\bar r$ local

Thm. Suppose that $\cdot\ p > 2n+1$ $\cdot$ assumption on $\bar\rho$.

(1) $\exists\ S_\infty = \mathcal{O}_E[[y_1, \dots, y_g]] \longrightarrow R_\infty = R_{\bar r}[[x_1, \dots, x_g]]$.

univ. def. ring

s.t. $R_\infty / (y_1, \dots, y_g) = R_{\bar\rho}$

and $\Pi_\infty \circlearrowleft GL_n(L)$.

Family of unitary Banach rep'ns $/R_\infty$.

$$\text{s.t. } \Pi_\infty[\underset{\text{torsion}}{(y_1,\dots,y_q)}] \simeq \hat{S}(U^p, E)_{\mathfrak{m}}.$$

$\mathfrak{m}$ = max'l ideal of $\mathcal{H} \simeq \bar{\rho}$.

compatible with

$$R_\infty/(y_1,\dots,y_q) \simeq R_{\bar{\rho}}.$$

[construction due to
Caraiani, Emerton, Gee, Geraghty, Paskūnas, Shin]

(ii) $\exists\ X(\bar{\rho}) \subseteq (\mathrm{Spf}(R_\infty))^{\mathrm{rig}} \times \hat{T}^n$

Zariski closed.

$(y, \delta) \in X(\bar{\rho}) \iff 0 \neq \mathrm{Hom}_T(\delta, J_B(\Pi_\infty[\mathfrak{p}_y]^{\mathrm{an}}))$

where $\mathfrak{p}_y \in \mathrm{Spec}\, R_\infty[\tfrac{1}{p}]$

corresponding to y.

s.t. $Y_{\bar{\rho}}$ gets identified with
union of irred cpnts of ϕ ~~$X_{\bar\rho}$~~!

$$X(\bar{\rho}) \times_{(\mathrm{Spf}(R_\infty))^{\mathrm{rig}}} \mathcal{X}_{\bar{\rho}}$$

$$\subseteq \mathcal{X}_{\bar{\rho}} \times \hat{T}^n.$$

conj $X(\bar\rho) \simeq X_{tri}(\bar r)$

union of irred cpnts

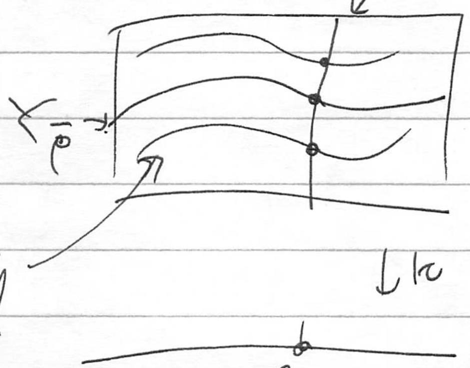
~~Picture~~

(iii) $X(\bar\rho) \hookrightarrow X_{tri}(\bar r) \times \mathfrak{U}^g$

$$\subseteq \underbrace{\mathfrak{X}_{\bar r} \times \mathfrak{U}^g \times \mathbb{U}^n}_{(\mathrm{Spf}\, R_\infty)^{rig}} .$$

<u>Picture</u>.

$\kappa^{-1}(w) \sim \begin{pmatrix} \text{pot crystalline} \\ \text{deform. ring} \end{pmatrix} \times \mathfrak{U}^g$



$X_{tri}(\bar r) \times \mathfrak{U}^g$.

eigenvariety for tame level U_i^p.

$\downarrow \kappa$

wt sp

w

locally alg. wt

"patching eigenvariety" ??