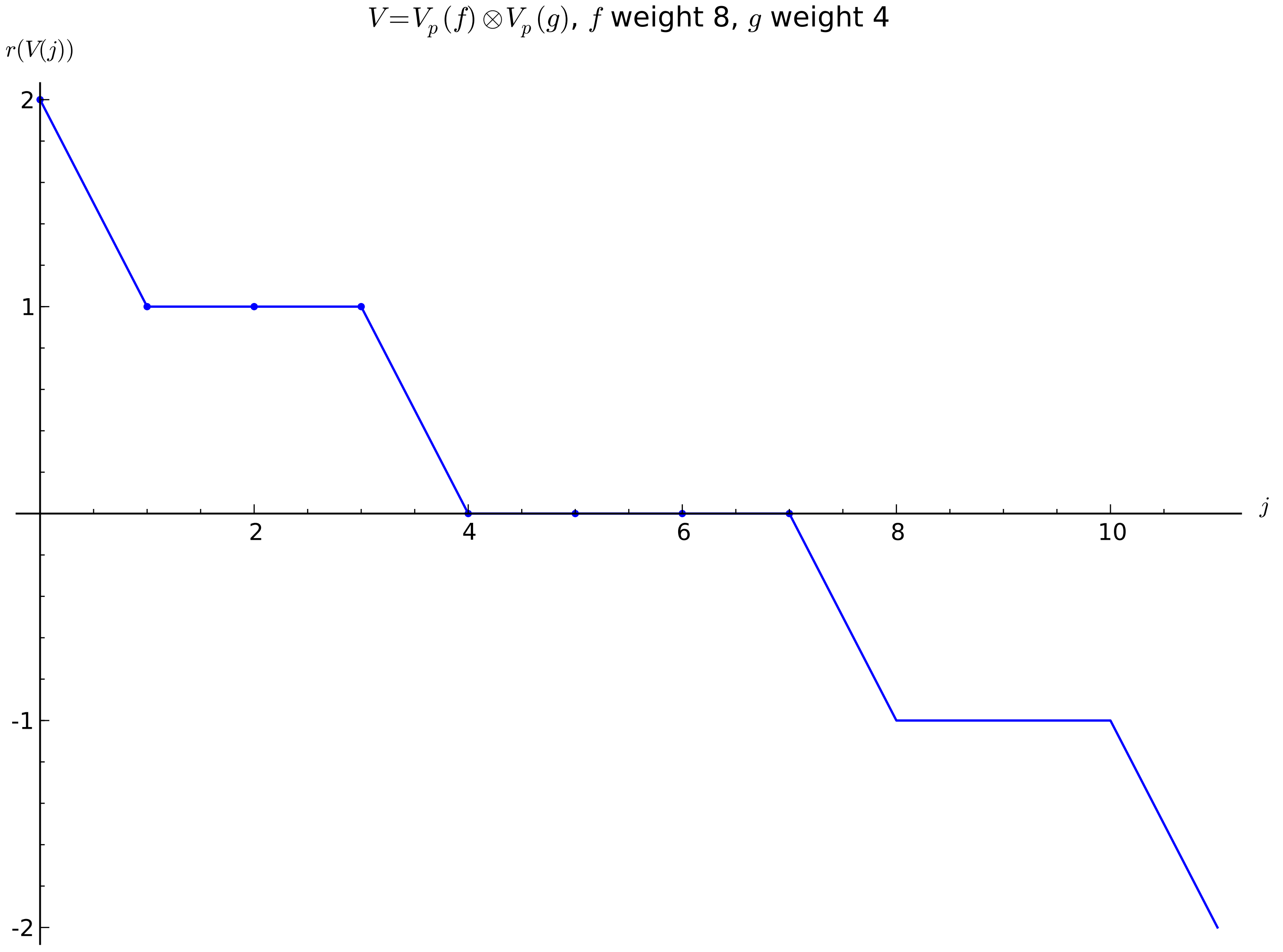
$V = V_p(f) \otimes V_p(g)$, $f$ weight 8, $g$ weight 4
$r(V(j))$
2
1
-1
-2
2
4
6
8
10
$j$

# Cases where $r(V(j)) = 1$

1.
   - $f$, $g$ mod forms of weight $k+2, k'+2$
   - $V = V_p(f) \otimes V_p(g)(j)$
   - $1 \leq j \leq 1 + \min(k, k')$
   - *(Lei–L.–Zerbes, Kings–L.–Zerbes)*
2.
   - $K$ real quadratic field
   - $f$ Hilbert mod form $/K$, weight $(k+2, k'+2)$
   - $V$ = Asai representation of $f$, $1 \leq j \leq 1 + \min(k, k')$
   - *(Lei–L.–Zerbes, in progress)*
3.
   - $F$ Siegel modular form for $\mathrm{GSp}_4/\mathbf{Q}$ of weights $(k, k')$, $k \geq k' \geq 3$
   - $V$ = spin representation of $F$ (4-dimensional)
   - $1 \leq j \leq k' - 2$
   - *(Lemma–L.–Zerbes, in progress)*

# Cases where $r(V(j)) = 1$

1.
   - $f, g$ mod forms of weight $k+2, k'+2$
   - $V = V_p(f) \otimes V_p(g)(j)$
   - $1 \le j \le 1 + \min(k, k')$
   - *(Lei–L.–Zerbes, Kings–L.–Zerbes)*
2.
   - $K$ real quadratic field
   - $f$ Hilbert mod form $/K$, weight $(k+2, k'+2)$
   - $V =$ Asai representation of $f$, $1 \le j \le 1 + \min(k, k')$
   - *(Lei–L.–Zerbes, in progress)*
3.
   - $F$ Siegel modular form for $GSp_4 / \mathbf{Q}$ of weights $(k, k')$, $k \ge k' \ge 3$
   - $V =$ spin representation of $F$ (4-dimensional)
   - $1 \le j \le k' - 2$
   - *(Lemma–L.–Zerbes, in progress)*

# Cases where $r(V(j)) = 1$

1. - $f, g$ mod forms of weight $k+2, k'+2$
   - $V = V_p(f) \otimes V_p(g)(j)$
   - $1 \le j \le 1 + \min(k, k')$
   - *(Lei–L.–Zerbes, Kings–L.–Zerbes)*
2. - $K$ real quadratic field
   - $f$ Hilbert mod form $/K$, weight $(k+2, k'+2)$
   - $V =$ Asai representation of $f$, $1 \le j \le 1 + \min(k, k')$
   - *(Lei–L.–Zerbes, in progress)*
3. - $F$ Siegel modular form for $\mathrm{GSp}_4/\mathbf{Q}$ of weights $(k, k')$, $k \ge k' \ge 3$
   - $V =$ spin representation of $F$ (4-dimensional)
   - $1 \le j \le k' - 2$
   - *(Lemma–L.–Zerbes, in progress)*