6-5-2014 David Loeffler Glennfest. 11:00AM.

Euler systems: new constructions, results, and conjectures.

§1. Intro.

V, p-adic rep'n of $G_{\mathbb{Q}}$.
unram. outside Σ ($p \in \Sigma$)

ES for V = classes

$$z_m \in H^1(\mathbb{Q}(\mu_m), V^*(1)) \quad \forall m$$

satisfying a compatibility relation involving L-factors of V.

Note. $H^1(\mathbb{Q}(\mu_m), V^*(1))$
is a module over
$$R_m = \mathbb{Q}_p[(\mathbb{Z}/m\mathbb{Z})^\times].$$

Why rank 2 BSD is difficult... ←

Observation (Gross):
If you want to construct a canonical element of a global Galois cohom. gp., that gp. had better to be of rank 1 (over the relevant base ring.)

Suppose V is odd ($\dim V^{c=+1} = \dim V^{c=-1} = d$, say, where c = complex conj.)

Then

$H^1(\mathbb{Q}(\mu_m), V^*(1))$ is "generically" of rank d / R_m.

(Euler-Poincaré + Leopoldt issue)

So if $d=1$, we're in good shape (e.g. Kato's ES).

What if $d > 1$?

Perrin-Riou: for $r \geq 1$, define rank r ES.

$= $ classes in $\bigwedge^r_{R_m} H^1(\mathbb{Q}(\mu_m), V^*(1))$

"higher rank ES"

Conjecture. $\exists$ rank d ES for V odd of dim $2d$.

Hard to construct!

§2. Local conditions

Idea: Pick out a canonical rank 1 submodule for classes to land in.

Do this with local conditions at p.

better to understand.

$$\begin{array}{ccc} H^1(\mathbb{Q}, V^*(1)) & \xrightarrow{loc_p} & H^1(\mathbb{Q}_p, V^*(1)) \\ & & \cup\!\! \mid \\ & & H^1_f(\mathbb{Q}_p, V^*(1)) \end{array}$$

Bloch-Kato.

"crystalline" classes

Fact. $\{ z \in H^1(\mathbb{Q}(\mu_m), V^*(1)) : loc_v\, z$ crystalline at $\forall v \mid p \}$.

has generic rank

$$d - \dim\left(\frac{D_{dR}(V)}{Fil^0}\right) =: r(V).$$

Conjecture For V coming from geometry (s.t. $r(V) \geq 0$)

$\exists$ ES for V of rank $r(V)$,

for which all classes are crystalline at p.

related to $L^{(r(V))}(V, 0)$

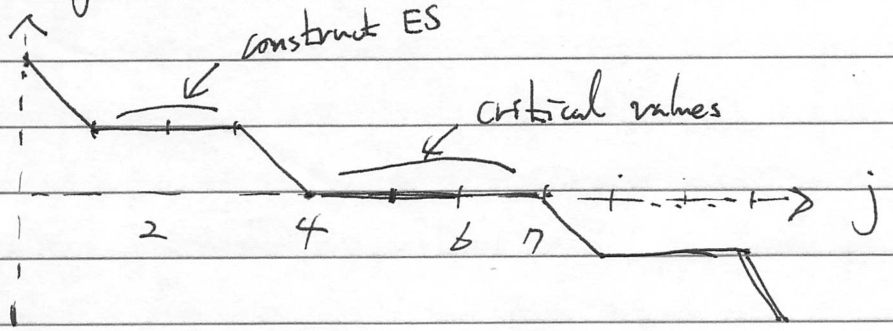
(note $L(V,s)$ vanishes to order $\geq r(V)$ at 0.)

Rank 0 E.S. = elts of R_m ← base ring.
related to L-values
= a p-adic L-ftn.

Can often choose j s.t. $r(V(j))$.
is 0 or 1.

(slide) $V = V_p(f) \otimes V_p(g)$ (wt 8, wt 4.)

$r(V(j))$



§3 Refinements.

Want to use these ES's to bound Selmer groups

<u>Def'n</u>. An <u>r-refinement</u> of V is a subspace $V^+ \subset V$ of dim $d-r$ (where V = $2d$-dim'l odd). ↑ stable by $G_{\mathbb{Q}_p}$ (not nec. by $G_{\mathbb{Q}}$).

- Say V^+ satisfies <u>the Panchishkin condition</u>

  if V^+ has all HT wt ≥ 1.

  and

  all H-T wt of $V/V^+ \leq 0$.

  (normalization: HT wt of $\chi_{cyc} = 1$)

  (This forces $r = r(V)$

  and

  V^+ is unique if exists.)

<u>Thm</u> (Rubin + ε · LLZ)

<u>If</u> V has a 1-refinement satisfying Panchishkin condition,

and

$\exists$ rank 1 ES for V that is crystalline at p

and

$z_1 \neq 0$

+ technical conditions,

<u>then</u>

$\mathrm{Sel}(\mathbb{Q}, V) = 0$.

Refinements make sense in families

(over rigid spaces / formal schemes)

<u>Conjecture</u>. If V, a families over X

V^+, 1-refinement over X,

(r−)

and

$\exists$ Zariski-dense set in X

where V^+ is Panchishkin,

then $\exists$ family of rank 1 ES for V

(r−1)

with

local condition given by V^+.