6-5-2014 Marco Seveso Glennfest 14:30pm.

Coleman vs. Colmez primitives on Mumford curves

Step. $W \simeq X \setminus D$ $\quad$ X complete nonsingular curve.

Fact $\exists W^\circ \subset W$ $\quad$ $D = \bigsqcup_{i=1}^{k} B[x_i, r_i]$

underlying affinoid domain

conn. cpnt. ~~$E \in (W \setminus W^\circ)$~~ $\quad$ $K = \mathbb{C}_p$

$$CC(W \setminus W^\circ) \longrightarrow \varprojlim_{W' \text{ conn subdomain}} CC(W \setminus W')$$

$A \in CC(W \setminus W^\circ)$ $\quad$ $A \simeq A(r, R)$ open annulus.

Def (Frobenius nbhd) (V, ϕ). (V: wide open)

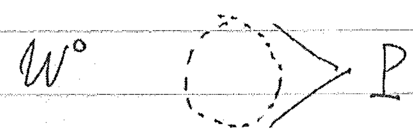
$$\begin{array}{ccc} W & & W \\ \cup & & \cup \\ V & \xrightarrow{\varphi} & V \\ \cup & \vdots \text{ lift} & \cup \\ W^\circ & \xrightarrow{\varphi} & W^\circ \end{array}$$

$A_V := A \cap V \xrightarrow{\varphi} A$

$A \in CC(W \setminus W^\circ)$

Problem. Single out a rigid enough class of loc. ~~const.~~ analytic fns for $dF = \omega$, ω rigid analytic ~~has~~ uniquely F upto constant.

Coleman's idea: Use Dwork's principle



$W^\circ \supset]P[$

$F \in \mathcal{L}(W^\circ)$ ← loc. analytic $\qquad P(\phi)\cdot F = 0$

$P(T)$ has not roots of unity.

$\rightsquigarrow F = 0$

$\exists n$, $\exists! \, x_P \in]P[$ s.t. $\varphi^n(x_P) = x_P$

$x \in]P[$, $\varphi^{mn}(x) \longrightarrow x_P$.

① $F(x_P) = 0 \qquad P(\phi) = \phi - a.$

$$F(x_P) = F(\phi^n(x_P)) = (\varphi^n \cdot F)(x_P)$$
$$= (a^n \cdot F)(x_P)$$
$$= a^n \cdot F(x_P)$$

$\Leftrightarrow (1 - a^n)\cdot F(x_P) = 0$

② F, loc. const.

$F|_V \equiv 0$ where $V \ni x_P$.

③ Use that for $x \in]P[$, then

$\varphi^{n \cdot m}(x) \longrightarrow x_P$.

to deduce that $F|_{]P[} \equiv 0$ for every $]P[$

(i) $dF = dG$

(ii) $P(\phi)(\underbrace{F-G}_{=H}) \in \mathcal{O}_{W^\circ}(W^\circ)$

$d(P(\phi) \cdot H) = P(\phi) \cdot dH = 0$.

$P(\phi) \cdot H = c = P(\phi) \cdot c$

$\Rightarrow P(\phi) \cdot (H - c) = 0$

$\Rightarrow H = c$.

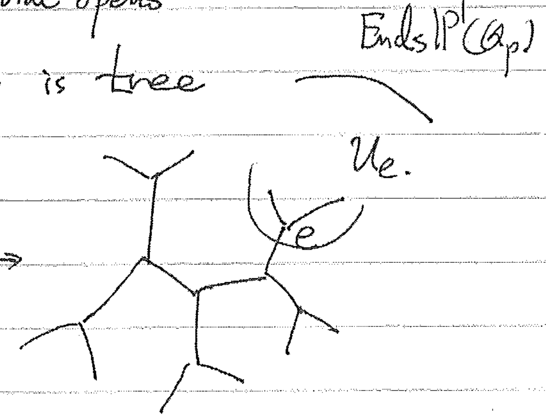
Thm (Coleman–de Shalit)
Given $\omega \in \Omega_W(W)$, given a Frob nbhd (V, φ),
$\exists!$ F_ω upto constant locally analytic function.
s.t. (i) $\omega = dF_\omega$
(ii) $P(\phi) \cdot F_\omega \in \mathcal{O}_V(V)$
(iii) $F_\omega|_{]P[}$ as above. $\rfloor$

$$F_\omega|_A \in \mathcal{O}_{A,\log}(A)$$
$$:= \mathcal{O}_A(A)[\log(f) : f \in \mathcal{O}_A(A)^\times].$$

<u>Step</u>. If I have given a curve s.t.
∃ covering by basic wide opens
whose ~~tree~~ nerve is tree
⇒ Can globalize.

$GL_2(\mathbb{Q}_p)$-equivariant

$$\mathfrak{h}_p = \mathbb{P}^1(\mathbb{C}_p) \setminus \mathbb{P}^1(\mathbb{Q}_p) \xrightarrow{v}$$



⇒ Get global primitive.

Teitelbaum $\mathcal{L}$-invariant.

Γ, arithmetic gp ⊂ $SL_2(\mathbb{Q}_p)$.

harmonic cocycles.

$$D_k^0(\mathbb{P}^1(\mathbb{Q}_p))^\Gamma \xrightarrow{R} C_{har}(\mathcal{E}, V_k)^\Gamma.$$

$$\mu \longmapsto R(\mu)(e)(P)$$

$$\downarrow \wr \qquad R(\omega)(e) = \mu(P \cdot \chi_{\mathcal{U}_e})$$

poly ↑ char fn. ↑

$\omega_\mu(\tau)(P) \in M_k(\Gamma, V_k)$ $\quad \overset{\shortparallel}{\mathrm{Res}_e}(\omega|\mathcal{U}_e)$ ← residue map

$$= \mu\left(\frac{P(z)}{z-\tau}\right) \cdot d\tau.$$

$$\mathrm{Char}(\mathcal{E}, V_k)^{\Gamma}$$

$$D_k^{\circ}(\mathbb{P}^1(\mathbb{Q}_p))^{\Gamma} \xrightarrow{\underline{P}\ \leftarrow (*)} H^1(\Gamma, V_k) \xleftarrow[\delta]{\sim} \mathrm{Char}(\mathcal{E}, V_k)^{\Gamma}$$

$$D_k^{\circ}(\mathbb{P}^1(\mathbb{Q}_p))^{\Gamma} \xrightarrow{\ \sim\ } M(\Gamma, V_k) \xrightarrow{\underline{P}} H^1(\Gamma, V_k) \quad \leftarrow \text{Coleman } \int.$$

$$\underline{P}(\omega)(\gamma) = \gamma(F_\omega) - F_\omega.$$

$$(*) \quad \underline{P}(\mu)(\gamma)(P) = \mu\left(\log\left(\frac{z-\gamma\tau}{z-\tau}\right)\cdot P(z)\right)$$

Def. $\underline{P} = \mathcal{L}\circ R$, $\mathcal{L} \in$ Hecke alg.

Families ~~of~~ mod. forms

and the cocycle ~~cocycle~~

$$\omega : \mathbb{Z}_p^\times \longrightarrow \mathcal{O}^\times \qquad \Omega \subset W$$

$\mathbb{Z}_p^\times \subset \mathbb{Q}_p^\times$, $\omega(p) = p^k$ on $k \in \Omega$, W = wt sp.

$N := \mathbb{Q}_p^2 \setminus \{0\}$.

$$D_k(\mathbb{P}^1(\mathbb{Q}_p)) \overset{\text{pull-back}}{\simeq} D_k(W)$$

$$\omega : \mathbb{Q}_p^\times \circlearrowright W \rightsquigarrow D(\mathbb{Q}_p^\times) \circlearrowright D(W)$$

$$D_\omega(W) = \mathcal{O}\hat{\otimes}_\omega D(W) \quad \text{naturally integrally}$$

Ash-Stevens $\quad A_\omega(W) = F$ loc. analytic

$$F(tw) = t^w F(w)$$

$$\begin{array}{ccc}
\mathcal{C}(V, D_\omega(W))^\Gamma & \longrightarrow & C(V, D_k(W))^\Gamma \\
\| & & \cup \\
\{c : V \longrightarrow D_\omega(W)\}. & & D_k(W)^\Gamma \\
 & & \cup \\
 & & D_k^\circ(W)^\Gamma. \\
\end{array}$$

$$\left(\begin{array}{ccc} \omega : \mathbb{Q}_p^\times & \longrightarrow & \mathcal{O}^\times \\ & \searrow^{k} & \downarrow \varphi \\ & & K^\times \end{array}\right) \qquad \begin{array}{c} \mu_{v,\omega} \\ \downarrow \\ \mu_{v,k} \end{array} \quad \text{s.t.} \quad \begin{array}{c} \mu \\ \| \\ \mu_{v,k} \end{array} \quad \forall v \in V.$$

$T \circlearrowright X$, $\pi : X \longrightarrow \cancel{T_x X} = T \backslash X = \overline{X}$, with $s : \overline{X} \to X$.

$\partial \in \mathrm{Der}(\varphi : \omega \longrightarrow k)$

$I_k := \ker(\varphi)$

$$\begin{array}{ccccc}
I_k \hat{\otimes} D(W) & \rightarrow & \mathcal{O} \hat{\otimes} D(W) & \xrightarrow{\partial \hat{\otimes} 1} & K \hat{\otimes} D(W) \\
\downarrow & & \circlearrowleft & & \downarrow \\
I_k \hat{\otimes}_\omega D(W) & & \xrightarrow{\exists \partial_\varphi} & & D_k(W) \\
\downarrow & & & & \\
D_\omega(W) & & \xrightarrow{\exists \partial_{\varphi, s}} & & D_k(W)
\end{array}$$

r : reductorm

Def. $C_\tau^{\mu_\omega,*}(\gamma)(P) = \partial_{s,\varphi}(\mu_{\omega,\gamma(\tau)})(P)$

$\tau \in \mathfrak{h}_p^{\,or}$ $\quad - \partial_{s,\varphi}(\mu_{\omega,\gamma r(\tau)})$

Prop. $C_\tau^{\mu_\omega,*} \in Z^1(\Gamma, V_k)$

and

$C_\tau^{\mu_\omega,*} = P_\tau(\mu)$ in $H^1(\Gamma, V_k)$.