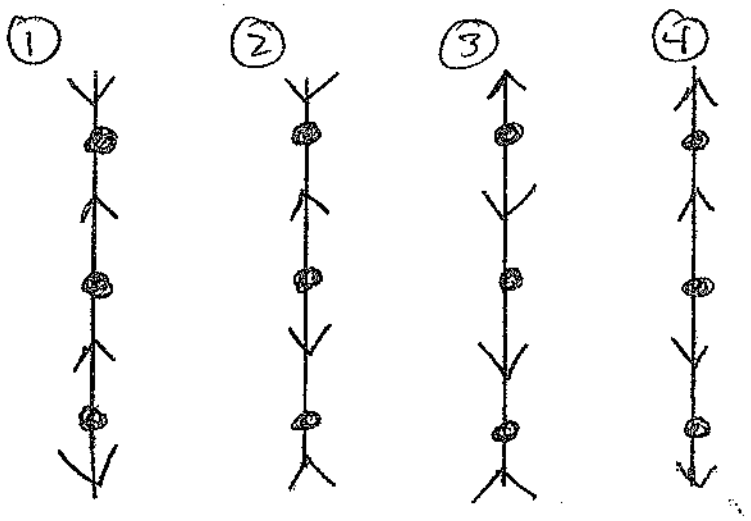
1. [16 Points] Here are four differential equations:

$$A.\ \frac{dy}{dt} = y(y^2 - 1) \qquad B.\ \frac{dy}{dt} = y^2(1 - y^2)$$

$$C.\ \frac{dy}{dt} = y|1 - y^2| \qquad D.\ \frac{dy}{dt} = y(1 - y^2)$$

And here are four phase lines. Match the number of the phase line with the letter of the associated differential equation. If no phase line corresponds to a given equation, write none next to that letter.



2. [20 Points] In an essay, describe Euler's method. First explain what the purpose of this method is. Then describe geometrically how Euler's method works; be sure to include a picture/graph to help explain this. Then give the formula for Euler's method. Be sure to explain exactly what all of the variables mean in this formula. And finally explain how this formula is used to produce the desired results.

3. [14 Points] Find the general solution of the differential equation

$$\frac{dy}{dt} - 3y = 5e^{3t}.$$

Show **ALL** work.

4. [15 Points] Find **all** equilibrium points for the differential equation

$$\frac{dy}{dt} = y\sin(y)$$

and determine which are sinks, sources, or nodes. Then sketch the phase line for this equation.

5. [15 Points] Sketch the bifurcation diagram for the family of differential equations $y' = Ay - y^3$ where A is a parameter. Describe in a couple of sentences where there is a bifurcation for this family and what happens at this bifurcation.

6. [20 Points] **Quickies.** Answers only. No partial credit.

   1. At which y-values does the Existence and Uniqueness Theorem fail for
   $$\frac{dy}{dt} = y^{1/3}.$$

   2. Sketch the phase line for $y' = -\pi$.

   3. Find all equilibrium solutions for the differential equation
   $$\frac{dy}{dt} = (t^2 - 1)(y^2 - 4)$$

   4. Sketch the slope field for
   $$\frac{dy}{dt} = \frac{2 - y}{2 + y}.$$

   5. **True or False:** The **general solution** of the differential equation
   $$\frac{dy}{dt} = y^2 \quad \text{is} \quad y(t) = \frac{-1}{t + C}.$$