F 15A

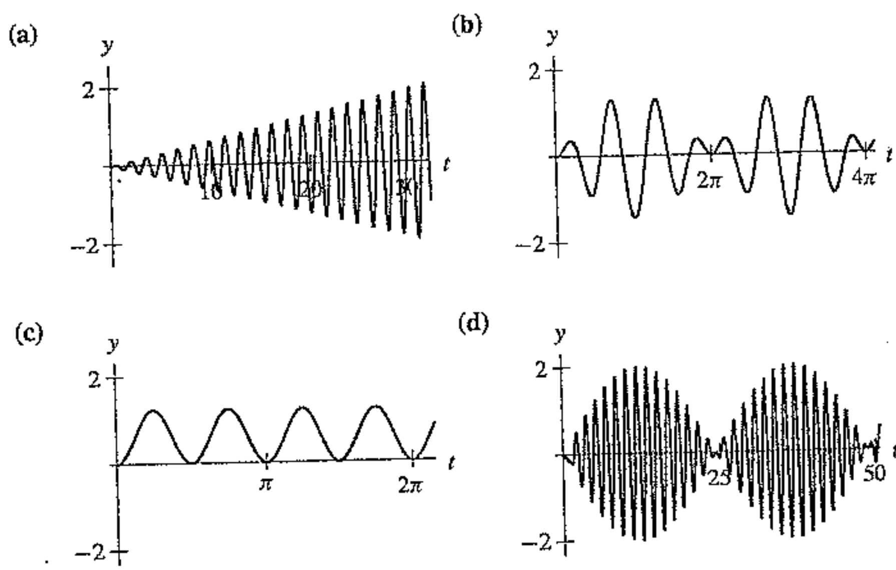
1. [25 Points] Find the solution of the second order differential equation

$$\frac{d^2y}{dt} + 4y = 4\cos(2t)$$

satisfying $y(0) = y'(0) = 0$ and sketch the graph of $y(t)$.

2. [20 Points] Six second-order equations and four $y(t)$ graphs are given below. For each $y(t)$ graph, determine the second-order equation for which $y(t)$ is a solution, and state in a sentence or two how you know your choice is correct. Each graph does correspond to one equation.

1. $y'' + 16y = 10$
2. $y'' + 16y = -10$
3. $y'' + 16y = 5\cos(3t)$
4. $y'' + 14y = 2\cos(4t)$
5. $y'' + 16y = \frac{1}{2}\cos(4t)$
6. $y'' + 2y' + 16y = \cos(4t)$



3. [30 points] Consider the nonlinear system of differential equations

$$\begin{aligned}\frac{dx}{dt} &= 3y^2 - x \\ \frac{dy}{dt} &= -x + 3y + 6.\end{aligned}$$

1. First find all equilibrium points for this system.
2. Determine the type (saddle, sink, etc.) of each of your equilibrium points using linearization. You need not compute eigenvectors.
3. Then draw the nullclines for this system.
4. Indicate which are the x- and y-nullclines.
5. Sketch the direction field in the regions between the nullclines.
6. Finally, sketch the solution curves in the phase plane for this system.

4. [25 points] Here is a system of differential equations that models the populations of a pair of competing species:

$$\begin{aligned}\frac{dx}{dt} &= x(-x - y + 40) \\ \frac{dy}{dt} &= y(-x^2 - y^2 + 2500).\end{aligned}$$

So you may asume that $x, y \geq 0$.

1. First sketch the nullclines for this system;
2. Find the equilibrium points (you need not determine the type of these points);
3. Sketch the phase portrait in the region $x, y \geq 0$;
4. Write a brief paragraph describing what happens to the population of these species when both $x(0)$ and $y(0)$ are positive.