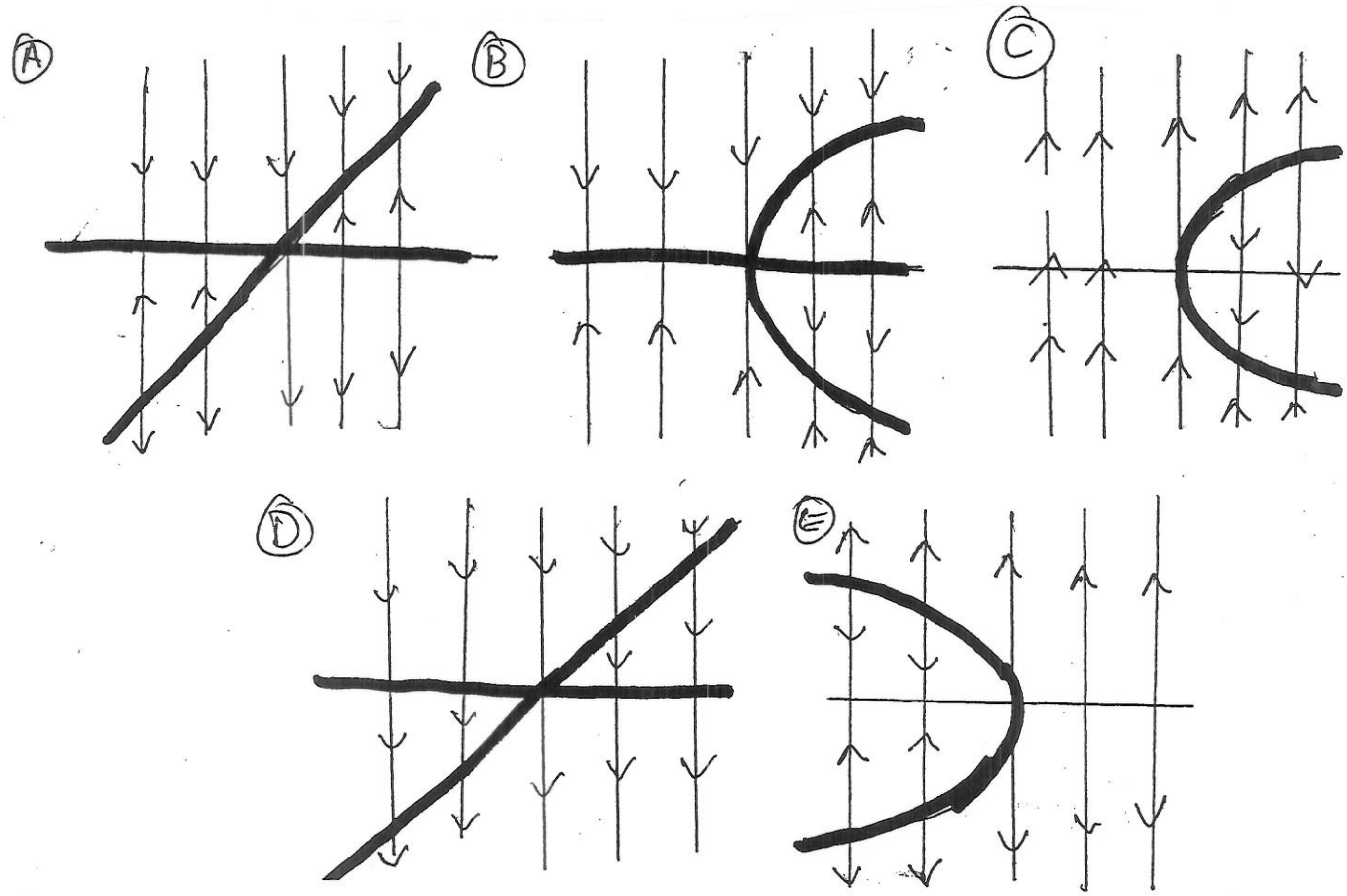
1. [15 Points] Bifurcations.



Above are five bifurcation diagrams A-E and below are five families of differential equations, each of which depends on a parameter A. Match the number of the differential equation to the letter of the bifurcation diagram. If no bifurcation diagram corresponds to a given equation, write NONE next to the given number.

1. $y' = Ay - y^3$ 2. $y' = y^2 - A$ 3. $y' = Ay - y^2$
4. $y' = A - y^2$ 5. $y' = A + y^2$

1 =	2 =	3 =	4 =	5 =

2. [20 Points] Linear Systems. Each of the following questions **may** have multiple correct answers. Place **each** correct letter in the box.

2A. Which of the following are eigenvalues for the matrix

$$\begin{pmatrix} 0 & -3 \\ 2 & 5 \end{pmatrix}$$

A. 0 B. 1 C. 2 D. 3 E. −2 F. None

Answer(s) =

2B. Which of the following are eigenvectors for the previous matrix

$$\begin{pmatrix} 0 & -3 \\ 2 & 5 \end{pmatrix}$$

$A.\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ $B.\begin{pmatrix} -3 \\ 2 \end{pmatrix}$ $C.\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ $D.\begin{pmatrix} -3 \\ 3 \end{pmatrix}$ $E.\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $F.\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ $G.$ None

Answer =

2C. Now sketch the phase plane for the linear system

$$Y' = \begin{pmatrix} 0 & -3 \\ 2 & 5 \end{pmatrix} Y$$

(i.e., same matrix as in the previous two problems).

2D. List **all** values of A for which the linear system

$$Y' = \begin{pmatrix} A & 1 \\ -1 & 0 \end{pmatrix} Y$$

undergoes a bifurcation.

Answer =

3. [12 Points] First order equations. Each of the following questions **may** have multiple correct solutions. Place the letter of **each** correct solution in the box.

3A. Which of the following functions are particular solutions of

$$y'' + 2y' + y = \sin(2t)$$

A. $e^{-t} + te^{-t}$

B. e^{-t}

C. $-\frac{3}{25}\sin(2t) - \frac{4}{25}\cos(2t)$

D. $-\frac{3}{25}\cos(2t) - \frac{4}{25}\sin(2t)$

E. $4e^{-t} - 6te^{-t} - \frac{3}{25}\sin(2t) - \frac{4}{25}\cos(2t)$

F. $6e^{-t} - 4te^{-t} - \frac{3}{25}\cos(2t) - \frac{4}{25}\sin(2t)$

Answer =

3B. The solution of the initial value problem

$$\frac{dy}{dt} + y = -2e^t,\ y(0) = 0$$

is:

$A.\ 2e^t - 2e^{-t}$ $B.\ -2e^t + 2e^{-t}$ $C.\ e^{-t} - e^t$
$D.\ e^t - e^{-t}$ $E.\ e^t - te^t - e^{-t}$ $F.$ None of these

Answer =

4. [15 Points] Laplace transforms.
4A. Which of the following is the inverse Laplace transform of

$$Y(s) = e^{-3s}\left(\frac{2}{s^2 - 1}\right)$$

$A.\ u_3(t)\sin(t-3)$ $B.\ u_3(t)(1/2)\sin(t-3)$
$C.\ u_3(t)(e^t - e^{-t})$ $D.\ u_3(t)(e^{t-3} - e^{-(t-3)})$
$E.\ u_3(t)(e^{-t} + e^t)$ $F.\ u_3(t)(e^{t+3} - e^{t-3})$
$G.$ None of these

Answer =

4B. Evaluate the following integral:

$$\int_0^5 (\delta_2(t) + 3u_4(t))dt$$

$A.\ 0$ $B.\ 1$ $C.\ 2$ $D.\ 3$ $E.\ 4$ $F.\ \infty$ $G.$ None of these

Answer =

4C. Compute the Laplace transform of

$$y(t) = \begin{cases} e^t & \text{if } t < 3 \\ 1 & \text{if } t \geq 3 \end{cases}$$

$A.\ \dfrac{1}{s-1} - \dfrac{e^{-3s}}{s-3} + e^{-3s}\dfrac{1}{s-3}$ $B.\ \dfrac{1}{s-1} - \dfrac{e^{-3s}}{s} + e^{-3s}\dfrac{1}{s}$

$C.\ \dfrac{1}{s-1} - e^{-3s}\dfrac{1}{s-1} + \dfrac{e^{-s}}{s}$ $D.\ \dfrac{1}{s-1} + e^{-3s}e^{3}\dfrac{1}{s-1}$

$E.\ \dfrac{1}{s-1} - e^{-3s}\dfrac{e^{3}}{s-1} + \dfrac{e^{-3s}}{s}$ $F.$ None of these

Answer =

5. [10 Points] Nonlinear systems.

5A. The equilibrium points for the system

$$\begin{aligned} x' &= y - x \\ y' &= x - y^2 \end{aligned}$$

are:

A. Sink and source B. Sink and saddle C. Both saddles
D. Saddle and source E. Spiral sink and saddle F. None of these

Answer =

5B. As time goes to infinity, the solution of the system

$$\begin{aligned} x' &= x^2 - 1 \\ y' &= 1 - y^2 \end{aligned}$$

that satisfies the initial condition $x(0) = y(0) = 0$ tends to:

$A.\ (1,1)$ $B.\ (1,-1)$ $C.\ (-1,1)$ $D.\ (-1,-1)$ $E.$ Infinity $F.$ None of these

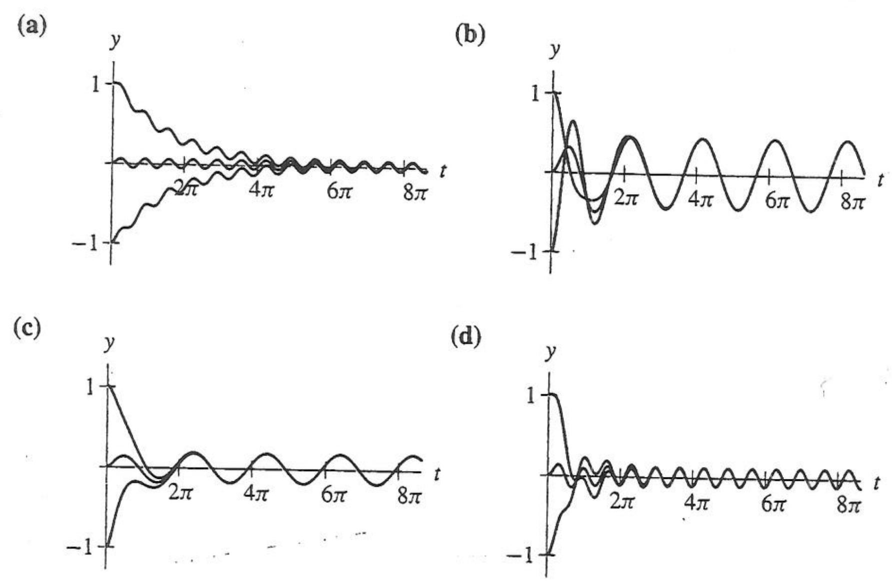
6. [12 Points] Second order equations. Six second-order equations and four $y(t)$-graphs are given below. The equations are all of the form

$$y'' + py' + qy = \cos(\omega t)$$

for various values of the parameters p, q, and ω. For each $y(t)$-graph, determine the second-order equation for which $y(t)$ is a solution.

(1) $p = 5, q = 2, \omega = 2$ (2) $p = 5, q = 1, \omega = 3$ (3) $p = 1, q = 1, \omega = 3$

(4) $p = 5, q = 3, \omega = 1$ (5) $p = 1, q = 3, \omega = 2$ (6) $p = 1, q = 3, \omega = 1$



a =	b =	c =	d =

7. [16 Points] Nonlinear systems

In this problem, you should show all of your work. This is not multiple choice. For the following system of differential equations, restrict attention to the first quadrant ($x, y \geq 0$). For this system:

1. find and determine the types of all equilibria;
2. sketch the nullclines;
3. then sketch a representative collection of solutions in the phase plane.

$$\begin{aligned} \frac{dx}{dt} &= x(-x - y + 70) \\ \frac{dy}{dt} &= y(-x^2 - y^2 + 2500) \end{aligned}$$

800. [1,000,000 Points] True/False. I am an engineer. __________________

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