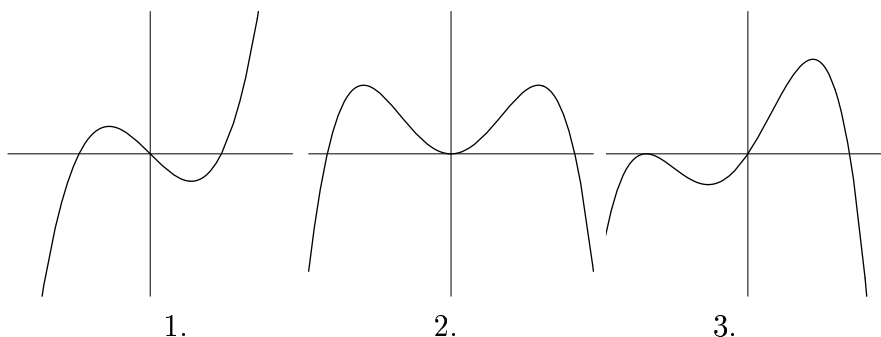
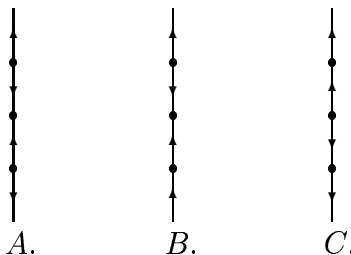


1. [15 Points] Here are three graphs of $F(y)$:



And here are three phase lines for a differential equation $y' = F(y)$. Match the letters of the corresponding phase lines with the numbers of the graphs. If no phase line corresponds to a given graph, write **NONE** next to the number of the graph.



2. [15 Points] Sketch the bifurcation diagram for the family of differential equations depending upon the parameter A :

$$\frac{dy}{dt} = y^3 - Ay.$$

3. [10 Points] On the screen before you you see a curve in the phase plane. Contrary to what is said on the screen, sketch only the $x(t)$ graph of this solution (not the $y(t)$ graph).

4. [15 Points] Find all equilibrium points for the differential equation

$$\frac{dy}{dt} = \sin^2(y)$$

and determine which are sinks, sources, or nodes.

5. [15 Points] Consider the curve in the phase plane given by

$$Y(t) = \begin{pmatrix} e^{-t} \\ 2e^{-t} \end{pmatrix}.$$

First sketch this curve in the xy -plane. Then sketch both its $x(t)$ and $y(t)$ graphs.

6. [15 Points] Find the general solution of the equation

$$\frac{dy}{dt} = -4y + 7.$$

7. [15 Points] In an essay, discuss Euler's method. Include a discussion of how Euler's method works. Be sure to give a precise formula for this method and tell what is the meaning of each term in this formula. Also, please include a picture detailing how Euler's method works in your essay and explain this picture in your essay.