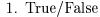
MA 471-671			
Fall 2004			
Final Exam			
Name:			

Do all problems and show all work.

MA 471-671

Fall 1997

Final Exam



- a. _____ The function $F(x) = x^2 2$ has infinitely many eventually fixed points.
- b. _____ The function $F(z) = z^2 1.705$ has a connected filled Julia set.
- c. _____ The Sierpinski triangle has fractal dimension $\log 2/\log 3$.
- d. _____ The complex derivative of the function $F(z) = |z|^2$ is 2|z|.
- e. _____ The Schwarzian derivative of $F(x) = e^x$ is constant.

- 2. Quickies. Answers only no partial credit.
- a. The functions $F(x) = x^2 + c$ has a _____ bifurcation at c = -5/4.
- b. Describe the behavior of all orbits of the function

$$F(x) = \begin{cases} -x & \text{if } x \ge 1\\ -2 + x & \text{if } x < 1 \end{cases}$$

c. Draw a sketch of the 3/7-bulb hanging off the main cardioid of the Mandelbrot set (with particular attention to the main antenna).

d. The doubling map of the unit circle $F(\theta) = 2\theta$ has a cycle of prime period 4 at: (give the "angle" (mod 1) of one such point).

e. For which values of c does the complex quadratic function $F(z)=z^2+c$ have two distinct fixed points?

f. List the Sarkovskii ordering of periods for continuous functions on the real line.

3.	Definitions. Give the precise definitions of each of the following.
	a. Filled Julia set.
	b. The function F is chaotic on a set S .

c.
$$H$$
 is a conjugacy between F and G .

d.
$$Q$$
 is a dense subset of the set T .

4. Describe in an essay complete with pictures all aspects of the saddle node bifurcation (real and complex) that occurs for the function $F(z) = z^2 + c$ when c = 1/4.

Continue your answer to question 4 here.

5. In an essay complete with pictures, discuss the Fundamental Dichotomy for the family of complex functions $F(z) = z^2 + c$. Be sure to include the relationship between this dichotomy and the Mandelbrot set.

Continue your answer to question 5 here.

- 6. Consider the shift map σ on the sequence space Σ_2 .
- a. Is the shift map continuous? If so, prove it. If not, explain why.
- b. Is the shift map onto? If so, prove it. If not, explain why.
- c. Is the shift map one-to-one? If so, prove it. If not, explain why.

Continue your answer to problem 6 here.

sequences that have If so, prove it. If no		dense in the

7.	Continued.	Continue your answer to question 7 on this page, or use it for scrap.	