Final Exam; Math 225 A1
December 20, 2002

This exam is a closed book, no notes, no "crib sheets" exam. Calculators are permitted. There are 10 problems on this exam – don't overlook those on the back of the page. The number of points that each problem is worth is printed next to the problem. You must show your work to receive credit for the problems. Good luck!

1. If $\underline{\mathbf{a}} = \langle 1, 1, -2\rangle$, $\underline{\mathbf{b}} = \langle 3, -2, 1\rangle$, and $\underline{\mathbf{c}} = \langle 0, 1, -5\rangle$, compute $\underline{\mathbf{a}} \cdot \underline{\mathbf{b}}$ and $\underline{\mathbf{a}} \cdot (\underline{\mathbf{b}} \times \underline{\mathbf{c}})$. (8 pts.)

2. Find an equation for the tangent plane to the graph of the function $z = y^2 \ln(x)$ at the point $(1, 4, 0)$. Make sure that your final answer is really in the form of the equation of a plane. (10 pts.)

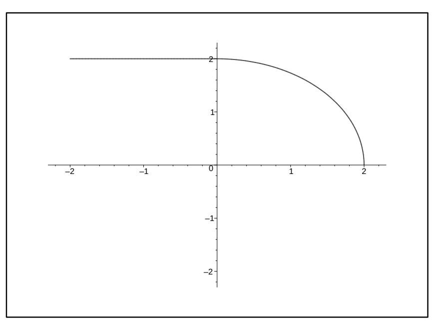
3. Evaluate the following double integral by reversing the order of integration: (10 pts.)

$$\int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3+1}\,dx\,dy \ .$$

4. If the acceleration of a particle is given by $\underline{\mathbf{a}}(t) = \langle 1, 2, 2t\rangle$ and if the initial velocity is $\underline{\mathbf{v}}(0) = \langle 0, 0, 0\rangle$ and initial position is $\underline{\mathbf{r}}(0) = \langle 1, 0, 1\rangle$, find the position of the particle $\underline{\mathbf{r}}(t)$ for all t. (10 pts.)

5. Write the integral that gives the arc length of the curve formed by the intersection of the cylinder $z^2 + y^2 = 4$ and the plane $x + y + z = 1$. *You do not have to evaluate this integral, however, you must write the integrand correctly in terms of the parameterization you choose for the curve and also choose the correct limits for the integral.* (10 pts.)

6. Find the work done by the force $\underline{\mathbf{F}} = x\underline{\mathbf{i}} - y\underline{\mathbf{j}}$ to move a particle along the path C formed by the quarter circle of radius 2 from $(2, 0)$ to $(0, 2)$ followed by the line segment from $(0, 2)$ to $(-2, 2)$. (10 pts.)
Hint: You may (or may not) want to use the fact that $\sin(2x) = 2\sin(x)\cos(x)$.



7. Let f be a scalar field and $\underline{\mathbf{F}}$ a vector field. State whether each of the following expressions is meaningful. If not, explain why. If so, state whether it is a scalar field or vector field. (12 pts.)

(a) curl(grad f)

(b) grad(div $\underline{\mathbf{F}}$)

(c) div(div $\underline{\mathbf{F}}$)

(d) (grad f) $\times$ (div $\underline{\mathbf{F}}$)

8. Use Green's theorem to rewrite $\oint_C x^2y^2dx + 4xy^3dy$ where C is the triangle with vertices $(0,0)$, $(1,3)$, $(0,3)$, traversed in the counterclockwise direction, as a double integral. You must write the double integral as an iterated integral with the correct limits, *but you do not have to evaluate these integrals.* (10 pts.)

9. Compute the flux of the vector field $\underline{\mathbf{F}} = \langle x, y, z^4 \rangle$ through the part of the cone $z = \sqrt{x^2+y^2}$, $0 \le z \le 1$ with downward orientation. (10 pts.)

10. Compute the flux of the vectorfield $\underline{\mathbf{F}} = \langle x - 3xy^2, y^3, z + x^2 + y^2 \rangle$ outward through the surface S, consisting of the top and sides of the cylinder $x^2 + y^2 \le 4$, $0 \le z \le 1$. (10 pts.)
Hint 1: It's a bad idea to try and evaluate this integral directly.
Hint 2: If you want to apply the divergence theorem, you must deal with the fact that S is not a closed surface (since the bottom of the cylinder is not part of S.)