Midterm 1; Math 225 A1
October 18, 2002

This exam is a closed book, no notes, no "crib sheets" exam. Calculators are permitted. There are ten problems on this exam – don't overlook those on the back of the page. The number of points that each problem is worth is printed next to the problem. Good luck!

1. Let $\underline{\mathbf{a}} = \underline{\mathbf{i}} + \underline{\mathbf{j}}$ and $\underline{\mathbf{b}} = \underline{\mathbf{i}} - \underline{\mathbf{k}}$. Compute $\underline{\mathbf{a}} \cdot \underline{\mathbf{b}}$ and $\underline{\mathbf{a}} \times \underline{\mathbf{b}}$. Here as usual, $\underline{\mathbf{i}}$, $\underline{\mathbf{j}}$, and $\underline{\mathbf{k}}$ are unit vectors in the x, y and z directions respectively. (10pts.)

2. State whether the following statements are true or false. If the statement is true, explain why. If it is false, either explain why or give an example showing that it is false. You must explain your answer correctly (or give a correct example illustrating why it is false) to receive credit.

   1. The linear equation $Ax + By + Cz + D = 0$ (with A, B, C, and D constants) represents a line in space. (6 pts.)

   2. There exists a function $f(x, y)$ with continuous second derivatives such that $f_x(x, y) = x + y^2$ and $f_y(x, y) = x - y^2$. (Hint: Think about what Clairaut's theorem tells you.) (6 pts.)

3. Find the length of the curve $\underline{\mathbf{r}}(t) = \langle 2t^{3/2}, \cos(2t), \sin(2t) \rangle$, for $0 \leq t \leq 1$. (10 pts.)

4. If $f(x, y) = \sin(2x + 3y)$, compute $f_y(-6, 4)$ and $f_{xy}(2, 1)$. (10 pts.)

5. Find the equation for the tangent plane to the graph of the function $f(x, y) = 4x^2 - y^2$ at $(1, 2, 0)$. (10 pts.)

6. Evaluate the following limit or show that it does not exist

$$\lim_{(x,y)\to(0,0)} \frac{2xy}{x^2 + 2y^2}.$$

(10 pts.)

Don't forget the problems on the back!

7. Find a parametric representation for the part of the plane $x + y = 2$ that lies inside the cylinder $y^2 + z^2 = 1$. Be sure to specify the domain of the parameters. (10 pts.)

8. An airplane is flying horizontally, in a straight line, over a flat field, at at altitude of 1000 meters and with a constant speed of 120 meters/s. When it is exactly above the "drop point" it releases a bag of flour which accelerates downward at a rate of 9.8 meters/s^2. How far from the drop point does the bag of flour land? (Hint: think about what the initial velocity and initial position of the bag of flour should be.) (12 pts.)

9. A contour map of a function f is shown below.

1. Use the contour map of f to make a rough sketch of the graph of f. Be sure to label the axes of your graph. (8 pts.)
2. Is $f_x(1.2, 0.2)$ positive or negative? How about $f_{xx}(1.2, 0.2)$? In both cases you must explain your answer – no credit will be given if you just write down "positive" or "negative". (8 pts.)