Homework Assignment 3 **Math 776**
Due Friday October 25

These are the problem numbers for the 2nd edition of Evan's book!

From the textbook please solve problems 19 and 20 on page 165, and problems 4 and 5 on page 306.

5. The equation

$$u_t + uu_x = \nu u_{xx} \ .$$

is called Burger's equation with viscosity. It is sometimes considered a one-dimensional model for the turbulent motion of fluids. Show that if one makes the change of variables

$$U(x,t) = \int_{-\infty}^{x} u(z,t)dz$$

that U satisfies the equation

$$U_t + \frac{1}{2}(U_x)^2 = \nu U_{xx} \ .$$

Now show that if one makes the further change of variables

$$w(x,t) = e^{-U(x,t)/(2\nu)}$$

w satisfies the heat equation! This change of variables is known as the Cole-Hopf transformation. Use the Cole-Hopf transformation to solve Burger's equation with viscosity with the initial condition $u(x,0) = e^{-x^2}$. (You may need to evaluate certain integrals numerically or with the aid of Maple or Mathematica.) Graph the solution for several different values of ν to see how shocks develop as $\nu \to 0$. (For more information about the Cole-Hopf transformation see section 4.5 of the text.)