Math 721A1, Homework #2
Differential Topology I

1. Guillemin and Pollack 3.6 (a) and (b), 3.8
2. Let $f : \mathbf{RP}^2 \to \mathbf{R}^3$ be the map
$$g([x,y,z]) := (yz, xz, xy)$$
where $x^2 + y^2 + z^2 = 1$. Show that g fails to be an immersion at 6 points (the image points are the points at distance $\pm\frac{1}{2}$ on each axis).
3. Let
$$S^n := \{\, (x_0, \ldots, x_n) \mid \sum_{i=0}^{n} (x_i)^2 = 1 \,\}$$
be the unit n-dimensional sphere in $\mathbf{R}^{n+1}$ and $\mathbf{RP}^n$ be n dimensional projective space. Find the rank of the canonical map $\pi : S^n \to \mathbf{RP}^n$ taking
$$(x_0, \ldots, x_n) \mapsto [x_0, \ldots, x_n].$$
4. (a) If M is a smooth manifold, a set $M_1 \subset M$ can be made into a k-dimensional submanifold of M if and only if around each point in M_1, there is a coordiante system (x, U) on M such that $M_1 \cap U = \{\, p \mid x^{k+1}(p) = \cdots x^n(p) = 0 \,\}$.
   (b) The subset M_1 can be made into a closed submanifold if and only if such coordinate systems exist around every point of M.