Math 721A1, Homework #3
Differential Topology I

1. Guillemin and Pollack 2.9, 11; 4.8, 11. (Notice that in Guillemin and Pollack, their map df is the map f_* introduced in class.)
2. Let $f : M^n \to N^m$ be a smooth map and let (x, U) and (y, V) be coordinate systems around p and $f(p)$ respectively. Show that
   (a) If $g : N \to \mathbf{R}$ (smooth map) then
   $$\frac{\partial(g \circ f)}{\partial x^i}(p) = \sum_{j=1}^{m} \frac{\partial g}{\partial y^j}(f(p)) \frac{\partial(y^j \circ f)}{\partial x^i}(p).$$
   (b) Show that
   $$f_*\left(\frac{\partial}{\partial x^k}\bigg|_p\right) = \sum_{j=1}^{m} \frac{\partial(y^j \circ f)}{\partial x^i}(p) \left.\frac{\partial}{\partial y^j}\right|_{f(p)}.$$
   More generally, express
   $$f_*\left(\sum_{k=1}^{n} a^k \left.\frac{\partial}{\partial x^k}\right|_p\right)$$
   in terms of the
   $$\left.\frac{\partial}{\partial y^j}\right|_{f(p)}.$$
3. Show the following:
   (a) Show that for any bundle $\pi : E \to B$, the map $s : B \to E$ with $s(p)$ the 0 vector of $\pi^{-1}(p)$ is a section.
   (b) Show that the n-plane bundle $\pi : E \to B$ is trivial if and only if there are n sections $s_1, \ldots, s_n$ which are everywhere linearly independent, i.e., $s_1(p), \ldots, s_n(p)$ are linearly independent for all $p \in B$.
   (c) Show that locally, every n-plane bundle has n linearly independent sections.
4. If $f : M \to N$ and f_* is the 0 map on each fiber then f is constant on each component of M.