Math 564, Final Exam, April 30, 2001
Due in my office (MCS 234) at 5pm on May 8, 2001.
[Please slide it under my door if I'm not there.]
Prof. Takashi Kimura

1. (10 points) Show that any finite union of compact subspaces of a topological space is compact.

2. (10 points) Problem #15 from Chapter 2 of our text.

3. (10 points) Let $f : X \to Y$ be a continuous function from a compact topological space X to a Hausdorff topological space Y. Show that f is a closed map.

4. (10 points) Let X be a set with two metrics d and d' which have the property that for all $x_1, x_2 \in X$,
$$d(x_1, x_2) > d'(x_1, x_2).$$
Let U be a nonempty subset of X. Prove that if U is an open set with respect to the metric topology on X from d' then U is an open set with respect to the metric topology on X from d.

5. (10 points) Let X be a topological space and x_0 be a given point in X. Furthermore, suppose that a subset U of X is an open set if and only if $x_0 \in U$. Can there exist a metric d on X such that its metric topology agrees with the topology on X? Prove your answer.

6. (10 points) Problem #17 from Chapter 2 of our text.

7. (10 points) Problem #34 from Chapter 4 of our text.

8. (10 points) Problem #1 from Chapter 5 of our text.

9. (10 points) Problem #8 from Chapter 5 of our text.

10. (10 points) Problem #25 from Chapter 5 of our text.