Math 564, Midterm Exam, April 2, 2001
Due in class on April 9, 2001
Prof. Takashi Kimura

Throughout, $\mathbf{R}$ is the set of real numbers, $\mathbf{Q}$ is the set of rational numbers and $\mathbf{Z}$ is the set of integers.

1. (10 points) Let
$$C_n := \{ (x, y) \in \mathbf{R} \mid x^2 + y^2 = \frac{1}{n^2} \}$$
and let $X := \cup_{n=1}^{\infty} C_n$ be given the subspace topology as a subset of $\mathbf{R}^2$. Is X closed? Prove your answer.

2. (10 points) Consider the set $\mathbf{R}$ of real numbers. Consider the subset
$$K := \{ \frac{1}{n} \mid n > 0 \text{ and } n \in \mathbf{Z} \}.$$
   (a) Let $\mathbf{R}$ be given the topology arising from the basis $\mathcal{B} := \{ [a, b) \mid a < b \}$. Find the closure of K in $\mathbf{R}$ in this topology.
   (b) Let $\mathbf{R}$ be given the topology arising from the basis $\mathcal{B}' := \{ (a, b] \mid a < b \}$. Find the closure of K in $\mathbf{R}$ in this topology.

3. (10 points) Let X be a topological space and let A be a connected subset of X. Is A connected? Prove your answer.

4. (10 points) Find the boundary and the interior each of the following subsets of $\mathbf{R}^2$:
   (a) $A := \{(x, y) \mid y = 0 \}$
   (b) $B := \{(x, y) \mid x > 0 \text{ and } y \neq 0 \}$
   (c) $C := A \cup B$
   (d) $D := \{(x, y) \mid x \in \mathbf{Q} \}$

5. (10 points) Problem #10 from Chapter 3 in our text.

6. (10 points) Problem #26 from Chapter 3 in our text.

7. (10 points) Let X and Y be topological spaces and suppose that $f : X \to Y$ is a continuous function. If x is a limit point of a subset A of X, is it necessarily true that $f(x)$ is a limit point of $f(A)$?

8. (10 points) Let f and g both be continuous functions $X \to CY$ where CY is the cone of Y. Show that f and g are homotopic.

9. (10 points) Problem #9 from Chapter 5 in our text.

10. (10 points) Problem #13 from Chapter 5 in our text.