Math 563A1, Final Exam, May 5, 1999
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This exam is due in my office at 5pm on Tuesday May 11. The exam is open book and you may work with others although you must write up the solutions yourself. Good luck!

1. (10 points) Exercise 2.4 from chapter 8 from our text.
2. (10 points) Exercise 3.19 from chapter 7 from our text.
3. (10 points) Let $\alpha : I \to S$ be a curve parametrized by arc length s, with nonzero curvature. Consider the parametrized surface
$$x(s,v) := \alpha(s) + vb(s)$$
where $s \in I$ ($I := [0,1]$) and $|v| < \epsilon$ for some $\epsilon > 0$ and where b is a binormal vector of α. Prove that if ϵ is small then $x(I \times (-\epsilon,\epsilon)) = S$ is a regular surface over which $\alpha(I)$ is a geodesic. Thus, every curve is a geodesic on a surface generated by its binormals.
4. (10 points) Consider the surface M in $\mathbf{R}^3$ given by
$$M := \{\, (x,y,z) \,|\, x^2 + y^{10} + z^6 = 1 \,\}.$$
Compute its Euler characteristic.
5. (10 points) Consider the hyperbolic "plane" H defined in example 4.3 on page 170. Let α be circle centered about $(0,0)$ in H of radius r where $r < 2$. Find the holonomy along α.