

Math 563A1, Final Exam, December 9, 2005
Prof. Takashi Kimura

This exam is due on Thurs Dec 15 at noon in my office MCS 234. Please slide it under my door if I'm not there.

The exam is open book and you may work with others although you must write up the solutions yourself. Good luck!

1. (20 points) Exercises Chapter 9: 4, 5, 8
2. (10 points) Let Σ be a surface without umbilic points and whose Weingarten matrix never vanishes. Let $x(u, v)$ be a surface chart of Σ such that its coordinate curves are lines of curvature. Let $U(u, v)$ be its associated unit normal. Consider

$$y(u, v) = x(u, v) + \rho_1 U(u, v)$$

and

$$z(u, v) = x(u, v) + \rho_2 U(u, v)$$

where $\rho_1 := 1/k_1$ and $\rho_2 := 1/k_2$ where k_1 and k_2 are the principal curvatures of Σ . The surfaces parametrized by $y(u, v)$ and $z(u, v)$ (call them Σ' and Σ'' resp.) are called the *focal surfaces* of Σ . Prove that

If $\frac{\partial k_1}{\partial u}$ and $\frac{\partial k_2}{\partial u}$ are nowhere zero then y and z are surface charts of (some open domain of) Σ' and Σ'' resp.

3. (10 points) Find the lines of curvatures and the asymptotic curves of $z = xy$.
4. (20 points) Exercises Chapter 11: 3, 4, 5.