Math 123, Practice Exam Questions for Exam #1, October 10, 2008

Unless explicitly stated, full credit will be awarded only if the solution is correct and the steps leading up to it are correct.

1. Calculate the following:

   (a)
   $$\lim_{x\to -1} \frac{x^2-4x-5}{x+1}$$

   (b)
   $$\lim_{x\to 3} \frac{5x^2}{2x-1}$$

   (c)
   $$\lim_{x\to 2^+} \frac{4-x^2}{|2-x|}$$

   (d)
   $$\lim_{x\to 1} \sqrt{\frac{2x^3-3x+5}{2-x}}$$

   (e)
   $$\lim_{x\to 3} f(x) \quad \text{where} \quad f(x) = \begin{cases} x^2 & \text{if } x > 3 \\ 8 & \text{if } x = 3 \\ 12-x & \text{if } x < 3 \end{cases}$$

   (f)
   $$\lim_{h\to 0} \frac{\sqrt[3]{8+h}-2}{h}$$

   (g)
   $$\lim_{x\to 3^-} \frac{x+3}{x^2-9}$$

   (h) The horizontal and vertical asymptotes of
   $$y = \frac{4-3x}{\sqrt{16x^2+1}}$$

   (i) $f'(x)$ where
   $$f(x) = \sin\left(x^{100}\right)$$

(j) $f'(x)$ where

$$f(x) = \sqrt{e^{2x} + 7x}$$

(k) $f'(x)$ where

$$f(x) = 10^{\cos x}$$

(l) $f'(x)$ where

$$f(x) = \frac{\ln x}{x}$$

(m) $f'(x)$ where

$$f(x) = x^{x^2}$$

(n) $f'(x)$ where

$$f(x) = \arctan(x^3)$$

2. Consider the function

$$f(x) = \begin{cases} x - c, & \text{if } x > 2; \\ 3x^2, & \text{if } x \leq 2 \end{cases}$$

where c is a real number.

(a) What value of c makes the function f continuous everywhere?

(b) Calculate $f'(7)$

(c) Calculate $f'(-1)$

(d) Calculate $f'(2)$

3. Compute the derivative of the following functions:

(a)

$$f(x) = 3x^5 - x^2 + 9$$

(b)

$$f(x) = \frac{2}{x^2} - 3\sqrt{x}$$

(c)

$$f(x) = \frac{x^2}{2x - 3}$$

(d)

$$f(x) = x^2 e^x$$

(e)

$$f(t) = t \sin t$$

4. Find the equation for the tangent line to the curve $y = f(x)$ where through the point $(1, -3)$ where
$$f(x) = x^8 - 4x.$$

5. Suppose an object is moving along the real line with its position at time t given by the function $s(t) = \frac{1}{3}t^3 - 3t^2 - 7t + 10$.
   (a) When is the object at rest?
   (b) When is the object moving to the right?
   (c) When is its acceleration positive?
   (d) Find the velocity of the object at time $t = 2$.

6. Find the equation for the tangent line to the curve given by the equation $\cos(xy) - 3y^3 = e^x + 1$ through the point $(0, -1)$.

7. Consider the graph of $y = f(x)$ on the next page.
   (a) Where is f undefined?
   (b) Where is f not continuous?
   (c) Where is f not differentiable?
   (d) On what interval(s) is f' positive? Where does f' vanish?
   (e) On what interval(s) is f concave down?
   (f) What are
      i. $$f'(6)$$
      ii. $$\lim_{x \to -5^+} f(x)$$
      iii. $$\lim_{x \to -1} f(x)$$
      iv. $$\lim_{x \to -\infty} f(x)$$