MA 124 Test 1

The bonus problem is more difficult than the other problems, please do as much as you can on problems 1 - 5 before spending time on the bonus.

You may find the following equations useful.

$$\cos 2x = \cos^2 x - \sin^2 x$$
$$\sin 2x = 2 \sin x \cos x$$

Problem 1

Evaluate the following integrals. Show all work.

1.

$$\int_{-\pi}^{\pi} \sin |\theta| \, d\theta$$

2.

$$\int \tan^2 x \cos^3 x \, dx$$

3.

$$\int_0^1 \ln x \, dx$$

4.

$$\int \sqrt{1 - x^2} \, dx$$

5.

$$\int_0^{\infty} \frac{1}{2^x} \, dx$$

Problem 2

Decide if the following integrals converge or diverge. For each, either compute the integral or demonstrate that it diverges.

1. $$\int_0^{\infty} \frac{1}{x^2}\, dx$$

2. $$\int_{-\infty}^{0} e^x\, dx$$

3. $$\int_0^{+1} \frac{1}{\sqrt{x^3}}\, dx$$

Problem 3

1. Find the following antiderivate

$$\int \frac{1}{(x+a)(x+b)}\, dx$$

2. Re-evaluate the antiderivative in part 1 when $b = a$. That is, find

$$\int \frac{1}{(x+a)(x+a)}\, dx$$

3. Can you set $b = a$ in your result from part 1 to get your result in part 2? Explain.

Problem 4

On the interval $0 \leq x \leq \pi$ consider the area between $y = \sin x$ and $y = 0$,

1. Find the volume created by rotating this area about the y-axis.

Problem 5

Given the function $f(x) = 2x - x^2$, prove the following

1.

$$0 \leq \int_0^2 f(x)\, dx \leq 2$$

2. For $x \geq 0$

$$\int_0^x f(t)\, dt \leq x$$

hint. $x = \int_0^x dt$

Bonus Problem

Mathematical induction is used to prove relationships true for all integers. Induction proofs involve the following two steps,

1. Show the relation is true for the smallest reasonable value of n.
2. Assume the relation is true for n and show that this implies the relation is then true for $n + 1$.

Use induction to prove the following relationship,

$$\int_{-\infty}^0 y^n e^y\, dy = (-1)^n n!$$