MA 124 Test 2

This exam has 4 problems and a bonus. Note that not all problems are the same number of points. You may find the following equations useful.

$$\cos 2x = \cos^2 x - \sin^2 x$$
$$\sin 2x = 2 \sin x \cos x$$

Problem 1 - 15 points

Find all solutions to the following differential equations. Check that your answers are correct.

1. $$\frac{dy}{dx} = x \cos x$$

2. $$\frac{dy}{dx} = y \ln x$$

3. $$2y^{3/2} + \frac{dy}{dx} = 0$$

Problem 2 - 30 points

An object with mass m is dropped from a height h. If the drag force is proportional to the square of the velocity, then the object's motion is modeled by the folloing equation,

$$m\frac{dv}{dt} = mg - kv^2$$

In this equation, v is the object's velocity ($v = dx/dt$), g is the constant acceleration due to gravity and k is a positive constant.

1. What is the object's terminal velocity?
2. Solve for $v(t)$ assuming $v(0) = 0$.
3. Show that $\lim_{t\to\infty} v(t)$ gives the terminal velocity.
4. How would you determine the time it takes the object to fall a distance h? Do not do the calculation.

Problem 3 - 30 points

At retirement, an individual has saved P_0 dollars and put the money in an account that compounds continuously at an annual interest rate r. If money is withdrawn at the rate of w dollars per year (modeled as continuous withdrawals), and $P(t)$ is the amount of money in the account at time t, then the following equation applies to $P(t)$.

$$\frac{dP}{dt} = rP - w$$

1. Solve the differential equation to find $P(t)$.
2. What is your solution if $w = 0$?
3. For what withdrawal rate w will $P(t)$ be a constant?
4. If w is greater than your answer in (3), will you run out of money, and if so, when?

Problem 4 - 25 points

A sealed cylindrical barrel with radius r is filled with water and lying perfectly horizontally on its side.

1. What is the hydrostatic force on one of the ends? You do not need to plug in numbers for ρ or g.
2. What is the hydrostatic force if the barrel is half submerged in water? That is, if the barrel is lying on its side in water with depth equal to the barrel's radius.

Bonus Problem - 10 points

In the following differential equation, c is a non-negative constant ($c \geq 0$).

$$\frac{dy}{dt} = y^2 - 2y + c$$

1. Find the equilibrium solutions. Remember to treat c as a parameter and discuss all cases.
2. Find $y(t)$ when c = 3/4