Suggestions for PS 14:

2. VII.12: If $\lambda < 0$, why is Ker $\lambda - A$ trivial? Show this implies Ran $\lambda - A$ is dense (by an earlier theorem). To show Ran $\lambda - A$ is closed let $(\lambda - A)x_n$ be Cauchy, and prove x_n is Cauchy by showing $|\langle(\lambda - A)x, x\rangle| \geq c\|x\|^2$.

4. Fourier spectrum: Show that $f_0(x) = e^{-x^2/2}$ is an eigenfunction. Consider the operator $D = \left(\frac{d}{dx} - x\right)$. Show that $\mathcal{F}(Df(x)) = -iD(\mathcal{F}f)(\xi)$. Thus show that $\mathcal{F}D^n e^{-x^2/2} = (-i)^n D^n e^{-x^2/2}$. The functions $D^n e^{-x^2/2}$ are multiples of the *Hermite functions.*