MA 717
M. Kon

PROBLEM SET 4
Due Thurs. Feb. 15

Lectures 7, 8

Starred problems are optional - they are typically not more difficult than other problems. They are worthwhile but will not be graded.

1*. Reed and Simon, II.1. Note here $\tilde{V}$ denotes the completion of V.

2*. Reed and Simon, II.2.

3. Reed and Simon, II.4 a; note the typo - the polarization identity should be

$$(x,y) = \frac{1}{4}\left\{\left(\|x+y\|^2 - \|x-y\|^2\right) + i\left(\|x+iy\|^2 - \|x-iy\|^2\right)\right\}.$$

4. Reed and Simon, II.6. Note that *linear subset* and *linear subspace* both refer to vector subpaces.

5. Reed and Simon, II.7

6. Reed and Simon, II.8.

7. Reed and Simon, II.10

8*. Reed and Simon, problem II.15. Recall $C_p^1[0,2\pi]$ denotes periodic functions with one continuous derivative.

9*. (a) On the interval $[-\pi,\pi]$ calculate the complex Fourier series $f(x) = \sum_{n=-\infty}^{\infty} c_n e^{inx}$ for the function

$$f(x) = \begin{cases} 1, & -1/2 \le x \le 1/2 \\ 0 & \text{otherwise} \end{cases} .$$

(b) Write down the partial sum from n = -1 to 1, and then using Euler's formula write it in terms of sin and cos functions.
(c) Using Euler's formula rewrite the full series as a real Fourier series in terms of cos nx and sin nx. Notice that when you reindex and combine terms your sum will need to have a constant term and a sum from n = 1 to ∞.
(d) Sketch the sums of the Fourier series in (c) after two and three terms have been summed (i.e. for the sum up to n = 1 and n = 2).

10. Reed and Simon, problem II.16. Show more specifically that any translation invariant measure on an infinite dimensional Hilbert space must be 0 on all balls sufficiently small, and ∞ on all balls sufficiently large.