Suggestions, PS 6

1. III.19: Can you use the open mapping theorem?

3*. III.22: For (a) try Corollary 3

4. III.25: First look at 2 dimensions, i.e., $L^1(X)$ and $L^\infty(X)$, where X is a measure space consisting of only two points. How many dimensions are these spaces? What do the unit balls look like? You can try two functions which are positive and have unit norm - what points on the plane do these two correspond to? For $L^1(\mathbb{R})$, try any two unit norm positive functions (you can relate this to the 2 point example). Does this work for $L^\infty(\mathbb{R})$? For (b) note the definition of uniform convexity involves just two vectors and their combinations, and we can restrict to the subspace spanned by them. In this case you need to show the circle in the plane is uniformly convex.