A parameterized surface in space is a set of points in space described by a (position) vector-valued function of the form

\[ \mathbf{r}(u, v) = x(u, v) \mathbf{i} + y(u, v) \mathbf{j} + z(u, v) \mathbf{k}, \]

where the functions \( x(u, v) \), \( y(u, v) \), and \( z(u, v) \) are defined on some region in the \( uv \)-plane.

**Example.** The elliptic cylinder

\[ x^2 + \frac{y^2}{4} = 1 \]

is parameterized by the function

\[ \mathbf{r}(\theta, z) = (\cos \theta) \mathbf{i} + (2 \sin \theta) \mathbf{j} + z \mathbf{k}. \]
In addition to rectangular coordinates in space, there are two other coordinate systems that are used frequently.

Cylindrical Coordinates

Cylindrical coordinates consist of polar coordinates in the $xy$-plane along with the usual rectangular coordinate $z$. Unlike polar coordinates, we often restrict our attention to the situation where $r \geq 0$.

Example. Use cylindrical coordinates to parameterize the hyperboloid of one sheet

$$x^2 + y^2 - z^2 = 1.$$
Spherical coordinates

Another 3D coordinate system that is often convenient to use is the spherical coordinate system.

Example. Parameterize the unit sphere using spherical coordinates.