More on triple integrals

Let's return to the triple integral that we were discussing on Friday.

Example. Evaluate

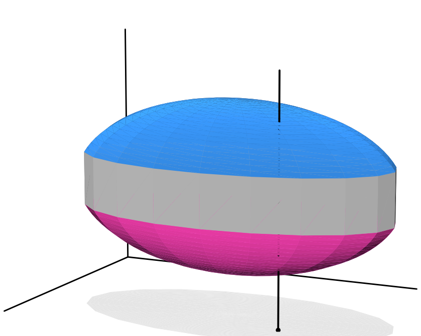
$$\iiint_Q z\,dV$$

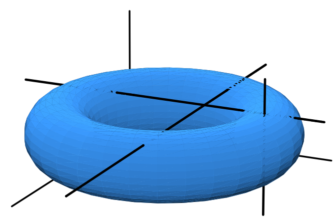
where Q is the region bounded by the cylinder $x^2 + z^2 = 9$, the plane $y + z = 3$, and the plane $y = 0$.

We evaluate a triple integral such as this one by repeated integration.

Definition. A solid region in space is z-simple if every vertical line that intersects the region enters and exits the region exactly once.

There are analogous definitions of x-simple and y-simple regions.





There is a version of Fubini's Theorem for triple integrals.

Theorem. (Fubini's Theorem) If Q is a z-simple region, then

$$\iiint_Q f(x,y,z)\,dV = \iint_{Q'} \left(\int_{g_1(x,y)}^{g_2(x,y)} f(x,y,z)\,dz \right) dA,$$

where Q' is the projection of Q onto the xy-plane and A is area in the xy-plane ($dA = dx\,dy$ or $dA = dy\,dx$).

Now back to the example at hand.

Example. Evaluate

$$\iiint_Q z \, dV$$

where Q is the region bounded by the cylinder $x^2 + z^2 = 9$, the plane $y + z = 3$, and the plane $y = 0$.

We can also view this region as x-simple and evaluate the integral that way. Try setting up the integral yourself. (Answer at the end of the handout.)

$$\int \int \quad \int \qquad z \, dx \, dz \, dy$$

Using cylindrical and spherical coordinates

Cylindrical and spherical coordinates can be used to simplify many triple integrals that possess radial or spherical symmetries.

Cylindrical coordinates (r, θ, z) are used when the region and function are best expressed in terms of polar coordinates in x and y.

Marilyn Vos Savant problem: Pick a sphere ... any sphere. Bore a perfect hole through the center in such a way that the remaining solid is 6 inches high. What is the volume that remains?

Limits of integration for the triple integral at the top of p. 3:

$$\int_0^6 \int_{-3}^{3-y} \int_{-\sqrt{9-z^2}}^{\sqrt{9-z^2}} z \, dx \, dz \, dy$$