Surfaces

The precise mathematical definition of a surface in space is technical and complicated. For our purposes, the following simpler statement will suffice.

Definition. A *surface* in space is a "two-dimensional" collection of points in space.

The meaning of the term "two-dimensional" is best illustrated by a few examples.

Examples.

1. Any plane $ax + by + cz = d$ is a surface.

2. The boundary of any solid region is a surface, e.g., the collection of points satisfying the inequality
$$x^2 + y^2 + z^2 \leq 1$$
has the spherical surface
$$x^2 + y^2 + z^2 = 1$$
as its boundary.

3. Take the equation of any curve in the xy-plane, e.g., the ellipse
$$2x^2 + y^2 = 1,$$
and plot all points (x, y, z) such that
$$2x^2 + y^2 = 1.$$
The result is a surface that is perpendicular to the xy-plane. This type of surface is called a (generalized) *cylinder*.

4. Surfaces of Revolution. Start with the graph of a function of one variable $z = f(y)$ in the yz-plane. Then we can revolve that curve around the y-axis in space. We get a surface of revolution.

5. The Mobius Band. Take a long and relatively thin strip of paper and attach the two short ends by a half twist.

6. Surfaces from torus knots

7. Carin Siegerman surface

8. Quadric Surfaces.

$$Ax^2 + By^2 + Cz^2 + Dx + Ey + Fz + G = 0.$$

One way to analyze a surface is by studying its traces.

Example. Consider the surface

$$x^2 + \frac{y^2}{4} - \frac{z^2}{9} = 1.$$

Suppose we intersect the surface with the plane $z = k$.

Given any equation for a quadric surface, I want you to be able to sketch it using the methods I described here. The slices are conic sections. You should know the names of the different types and what the differences are. There are six types.

(a) Ellipsoid

(b) Elliptic Paraboloid

(c) Elliptic Cone

(d) Hyperboloid of One Sheet

(e) Hyperboloid of Two Sheets

(f) Hyperbolic Paraboloid

See p. 691 of your textbook.