Quadric surfaces

$$Ax^2 + By^2 + Cz^2 + Dx + Ey + Fz + G = 0.$$

We start by studying the traces (cross sections) in various planes.

Example. Consider the surface

$$x^2 + \frac{y^2}{4} - \frac{z^2}{9} = 1.$$

Suppose we intersect the surface with the plane $z = k$.

Given any equation for a quadric surface, I want you to be able to sketch it using the methods I described here. The slices are conic sections. You should know the names of the different types and what the differences are. There are six types.

1. Ellipsoid
2. Elliptic Paraboloid
3. Elliptic Cone
4. Hyperboloid of One Sheet
5. Hyperboloid of Two Sheets
6. Hyperbolic Paraboloid

See p. 682 of your textbook.

Parametric Surfaces and 3D Coordinate Systems

A parameterized surface in space is a set of points in space described by a (position) vector-valued function of the form

$$\mathbf{r}(u,v) = x(u,v)\,\mathbf{i} + y(u,v)\,\mathbf{j} + z(u,v)\,\mathbf{k},$$

where the functions $x(u,v)$, $y(u,v)$, and $z(u,v)$ are defined on some region in the uv-plane.

Example. The elliptic cylinder

$$x^2 + \frac{y^2}{4} = 1$$

is parameterized by the function

$$\mathbf{r}(\theta,z) = (\cos\theta)\,\mathbf{i} + (2\sin\theta)\,\mathbf{j} + z\,\mathbf{k}.$$

In addition to rectangular coordinates in space, there are two other coordinate systems that are used frequently.

Cylindrical Coordinates

Cylindrical coordinates consist of polar coordinates in the xy-plane along with the usual rectangular coordinate z. Unlike polar coordinates, we often restrict our attention to the situation where $r \geq 0$.

Example. Use cylindrical coordinates to parameterize the hyperboloid of one sheet

$$x^2 + y^2 - z^2 = 1.$$