Functions of many variables

Many things are best modeled by functions of more than one variable.

Example. The cost of producing an item is a function of the wage rate, the number of hours it takes to produce the item, and the cost of materials.

Visualizing functions of more than one variable

For a function of one variable $y = f(x)$, we tend to visualize it by drawing its graph in the xy-plane. For a function of more than one variable, there are other ways to visualize it.

Example. Consider the function $h(x, y) = 4x^2 + y^2$. What kind of surface is its graph?

Definition. Given a function $f(x, y)$ of two variables, its level set of level K is the set of all points (x, y) such that
$$f(x, y) = K.$$

We see functions that are displayed in terms of their level sets all of the time. A typical example is Figure 6 on p. 742 of your textbook. It displays sea-level temperatures in January. We are also used to visualizing altitude in topographic maps.
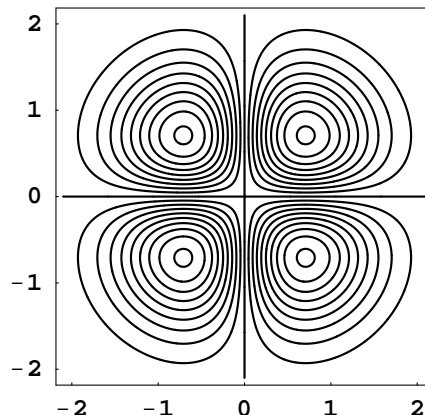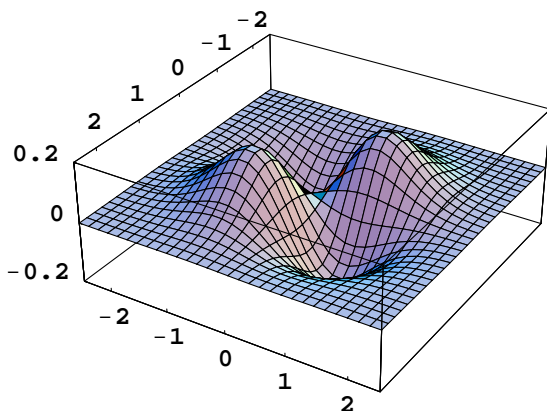
Example. For the function $h(x, y) = 4x^2 + y^2$ above, what can we say about its level sets?

On the web site, you will find a link to a particularly nice animation of level sets by Professor Lou Talman.

Computers are especially good at drawing contour maps.

Example. Consider the function
$$f(x, y) = \frac{-xy}{e^{x^2 + y^2}}.$$

See p. 745 of your textbook.

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Functions of three variables

For a function $f(x, y, z)$ of three variables, its graph would be the graph of

$$w = f(x, y, z).$$

Example. Sketch the level sets of the function

$$P(x, y, z) = x + y + 10z.$$

Example. Sketch the level sets of the function

$$f(x, y, z) = x^2 + y^2 - z^2.$$

MA 225 Exam Logistics

1. Bring pen/pencil and id. You may use your calculator if you wish. If you have a calculator that does symbolic derivatives/integrals, you should make sure that your answer shows that you could do the problem without your calculator.

2. Closed book exam. No extra papers. No ipods, cell phones, etc.

3. Exam will start promptly at 10:00 and end at 10:50.

4. We will collect exams by moving up the aisles. You must pass in your exam when we arrive at your aisle. Please remain seated and quiet until we collect the exams from your aisle.

5. Five minute rule will be in effect: No one will be allowed to leave the exam between 10:45 and 10:50. Use those 5 minutes to check your work.

6. Seating will be assigned before the exam starts.

7. If you have a question, raise your hand. Stay seated.

8. Go to the bathroom before the exam starts.