Three-dimensional coordinate systems and more parametric surfaces

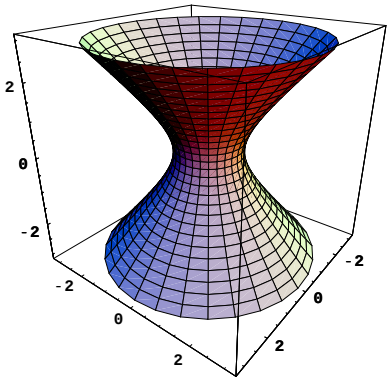
Today we consider nonrectangular three-dimensional coordinate systems, and we use these coordinates to help us parametrize certain surfaces. There are two such coordinate systems that we use frequently in this course.

Cylindrical Coordinates

Cylindrical coordinates consist of polar coordinates in the xy-plane along with the usual rectangular coordinate z. Unlike polar coordinates, we often restrict our attention to the situation where $r \geq 0$.

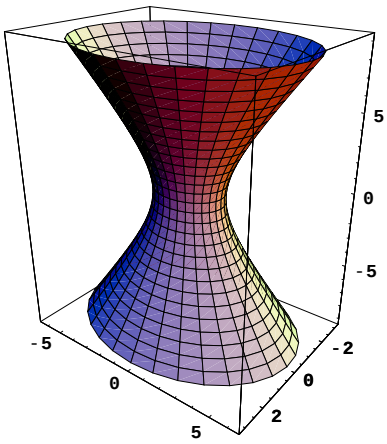
Example. Use cylindrical coordinates to parametrize the circular hyperboloid of one sheet

$$x^2 + y^2 - z^2 = 1.$$



Example. Use cylindrical coordinates to parametrize the elliptical hyperboloid of one sheet

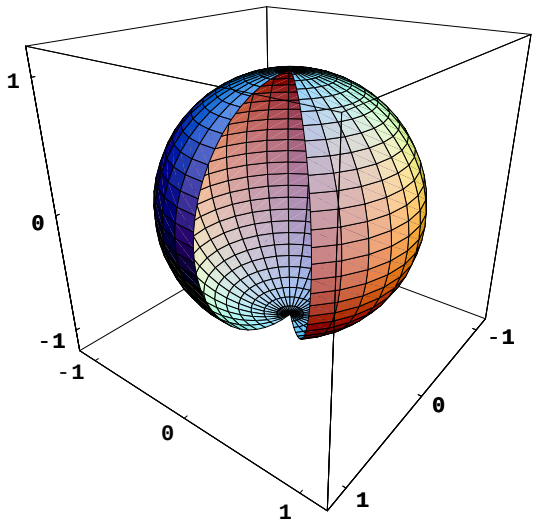
$$x^2 + \frac{y^2}{4} - \frac{z^2}{9} = 1.$$



Spherical coordinates

Another three-dimensional coordinate system that is often convenient to use is the spherical coordinate system.

Example. Parametrize the unit sphere using spherical coordinates.

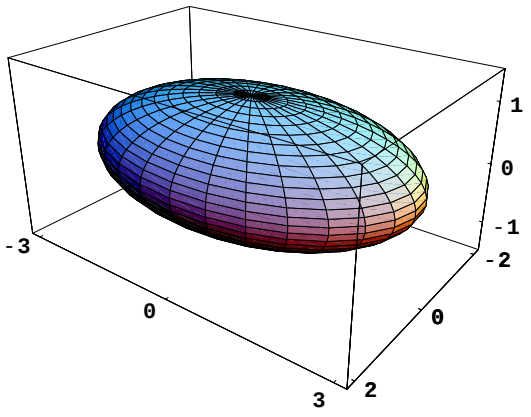


We can also parametrize ellipsoids and plot many unusual surfaces with spherical coordinates.

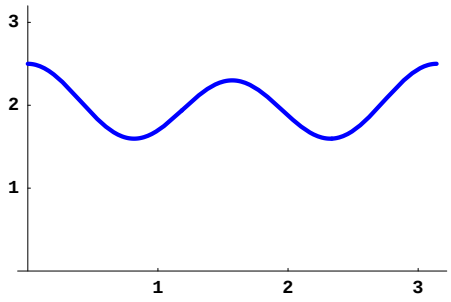
Example. Parametrize the ellipsoid

$$\frac{x^2}{4} + \frac{y^2}{9} + z^2 = 1$$

using spherical coordinates.



Example. We can use spherical coordinates and the function



to produce a top.