MA 225 Sample Examinations

Here are the four examinations that were given in my MA 225 class in the Fall of 2006. You should use them to get an idea of the format of a typical test and to see the types of questions I ask. *You should not assume that the test questions this semester will be on the same topics.* In fact, you are always responsible for *all* of the material that we cover in class as well as *all* of the designated material from your text. The best way to study for my exams is to be sure that you are very comfortable with the homework assignments and the examples that I present in class. My tests often vary in difficulty (as you can see here), and your grade for the examination will be determined by a curve that will be announced in class after the examination is graded.

MA 225	Exam 1A	October 4, 2006

Name: ______________________________ Last five digits of ID Number: __________

Discussion Section (circle yours): M 12–1 M 1–2 M 4–5 T 2:30–3:30 T 3:30–4:30

Directions: Please do all of your work in this exam booklet and make sure that you cross out any work that we should ignore when we grade. **Books and extra papers are not permitted.** If you have a question about a problem, please ask. Remember: answers that are written logically and clearly will receive higher scores. There are 5 questions on 5 pages (not counting this cover page). Please make sure that you have all 5 pages of questions.

Do not write in the following box:

PROBLEM	POSSIBLE	SCORE
Name, disc sec	3	
1	21	
2	18	
3	20	
4	18	
5	20	
TOTAL	100	

1. (21 points) Calculate:

   (a) The length of the projection of the vector $\mathbf{A} = \mathbf{i} + \mathbf{j} - 2\mathbf{k}$ onto the vector $\mathbf{B} = -2\mathbf{i} - \mathbf{j} + \mathbf{k}$.

   (b) A vector equation for the line that contains the point $(2, -1, 3)$ and is perpendicular to the plane $x + 3y - 2z = 4$.

   (c) The area of the triangle in space with vertices $P = (0, 1, 0)$, $Q = (4, 1, -2)$, and $R = (5, 3, 1)$.

2. (18 points) Let
$$\mathbf{r}(t) = (3t^2)\,\mathbf{i} + (te^t)\,\mathbf{j} + (\ln t)\,\mathbf{k},$$
and suppose that $\mathbf{r}(t)$ is the position function of a particle moving through space. Calculate the velocity, acceleration, and speed of the particle when it is at the point $(3, e, 0)$.

3. (20 points) Find the area of the region in the first quadrant that is inside the circle $r = 3\cos\theta$ and below the line $y = x$. (The half-angle formula for cosine is $\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$.)

4. (18 points) The two lines

$$\mathbf{L}_1(t) = (1+t)\,\mathbf{i} + (1-2t)\,\mathbf{j} + (3t)\,\mathbf{k}$$

and

$$\mathbf{L}_2(t) = (2+t)\,\mathbf{i} + (11+2t)\,\mathbf{j} + (-3+t)\,\mathbf{k}$$

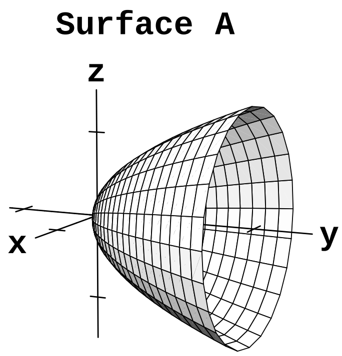
intersect.

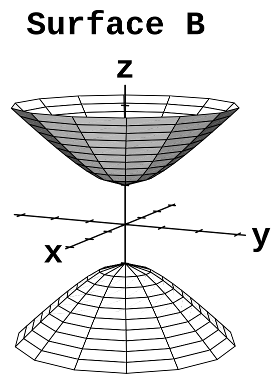
(a) Where do they intersect?

(b) Calculate the angle of intersection.

(c) Find an equation for the plane that contains these two lines.

5. (20 points) Here are two surfaces in space:





Here are 6 equations of surfaces:

1. $z^2 - x^2 - y^2 = 1$
2. $x^2 - y^2 + z^2 = 0$
3. $2x + y - z = 3$
4. $x^2 - y + z^2 = 0$
5. $x^2 + y^2 - z = 0$
6. $x^2 + y^2 - z^2 = 1$

For each surface, pick the equation that describes it. Provide a brief justification for your choice. **You will not receive any credit unless you provide a valid justification.**

(a) The equation for surface A is ____. My reason for choosing this answer is:

(b) The equation for surface B is ____. My reason for choosing this answer is:

MA 225	Exam 2A	November 1, 2006

Name: ____________________________ Last five digits of ID Number: __________

Discussion Section (circle yours): M 12–1 M 1–2 M 4–5 T 2:30–3:30 T 3:30–4:30

Directions: Please do all of your work in this exam booklet and make sure that you cross out any work that we should ignore when we grade. **Books and extra papers are not permitted.** If you have a question about a problem, please ask. Remember: answers that are written logically and clearly will receive higher scores. There are 5 questions on 5 pages (not counting this cover page). Please make sure that you have all 5 pages of questions.

Do not write in the following box:

PROBLEM	POSSIBLE	SCORE
Name, disc sec	3	
1	18	
2	21	
3	18	
4	20	
5	20	
TOTAL	100	

1. (18 points) Consider the surface

$$2x^2 - xy + 3y^2 + 4z^2 + yz = 14.$$

(a) Find an equation for the tangent plane to this surface at the point $(1, 2, -1)$.

(b) Find an equation for the normal line to this surface at the point $(1, 2, -1)$. (You may express your answer in any of the three forms that we have discussed—vector, parametric, or symmetric.)

2. (21 points) Suppose that the gradient of a function $f(x,y)$ is given by

$$\nabla f(x,y) = (e^{x^2} + y^2)\,\mathbf{i} + (2xy + 3y^2)\,\mathbf{j}.$$

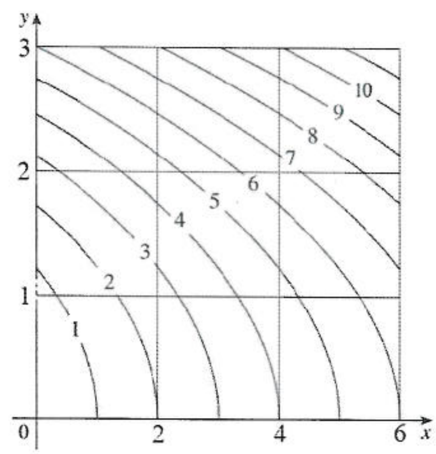
(a) Calculate $\dfrac{\partial^2 f}{\partial y^2}(3,2)$.

(b) Given the vector-valued function $\mathbf{P}(t) = (3t^2 - 4t)\,\mathbf{i} + 3t^2\,\mathbf{j}$, calculate

$$\left.\frac{d}{dt} f(\mathbf{P}(t))\right|_{t=1}$$

(c) Calculate the value of the largest directional derivative of f at the point $(1,-2)$.

3. (18 points) Here is a contour map for a function $f(x, y)$ on the rectangle $R = [0, 6] \times [0, 3]$.



(a) Use the Midpoint Rule with three subdivisions on each side to estimate the average value of $f(x, y)$ over the rectangle R. (Make sure that you show the intermediate steps in your calculation.)

(b) Give a brief explanation of why your estimate makes sense.

4. (20 points) Find the volume of the solid bounded by the planes $x = 0$, $y = 0$, $z = 0$, and $x + 2y + z = 2$ by evaluating a double integral.

5. (20 points) Find the volume of the largest rectangular box with edges parallel to the axes that can be inscribed in the ellipsoid

$$9x^2 + 36y^2 + 4z^2 = 36.$$

MA 225	Exam 3A	December 6, 2006

Name: ______________________________ Last five digits of ID Number: __________

Discussion Section (circle yours): M 12–1 M 1–2 M 4–5 T 2:30–3:30 T 3:30–4:30

Directions: Please do all of your work in this exam booklet and make sure that you cross out any work that we should ignore when we grade. **Books and extra papers are not permitted.** If you have a question about a problem, please ask. Remember: answers that are written logically and clearly will receive higher scores. There are 5 questions on 5 pages (not counting this cover page). Please make sure that you have all 5 pages of questions.

Do not write in the following box:

PROBLEM	POSSIBLE	SCORE
Name, disc sec	3	
1	20	
2	19	
3	18	
4	20	
5	20	
TOTAL	100	

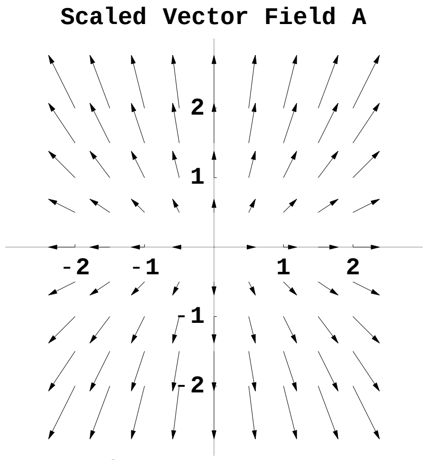
1. (20 points) Calculate the line integral

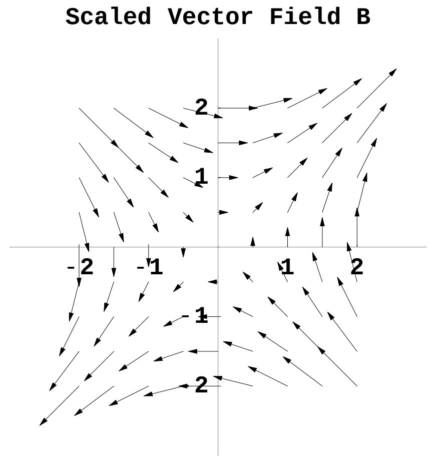
$$\int_C (x+y)\,dx + 2x\,dy + xy\,dz,$$

where the curve C consists of a line segment that goes from $(1,0,1)$ to $(2,3,1)$ followed by a line segment that goes from $(2,3,1)$ to $(2,5,2)$.

2. (19 points) Find the surface area of the part of the paraboloid $x = y^2 + z^2$ that lies inside the cylinder $y^2 + z^2 = 9$.

3. (18 points) Here are two gradient vector fields:





Here are 6 functions:

1. $f(x,y) = x^2 + 2y^2$
2. $f(x,y) = x^2 - y^2$
3. $f(x,y) = -xy$
4. $f(x,y) = 2x^2 + y^2$
5. $f(x,y) = xy$
6. $f(x,y) = y^2 - x^2$

For each vector field, pick the function whose gradient is that field. Provide a brief justification for your choice. **You will not receive any credit unless you provide a valid justification.**

(a) The function for field A is ____. My reason for choosing this answer is:

(b) The function for field B is ____. My reason for choosing this answer is:

4. (20 points) Use Green's Theorem to calculate the line integral

$$\oint_C (3\sqrt{x}+y^3)\,dx+(x+2\sqrt{y})\,dy,$$

where C is the curve in the xy-plane that follows the arc of $y=\sin x$ from $(0,0)$ to $(\pi,0)$ and the line segment from $(\pi,0)$ to $(0,0)$.

5. (20 points) Evaluate the triple integral

$$\iiint_T 24xz\,dV,$$

where T is the solid tetrahedron with vertices $(0,0,0)$, $(0,1,0)$, $(1,1,0)$, and $(0,1,1)$.

MA 225	Final A	December 15, 2006

Name: ______________________________ Last five digits of ID Number: ___________

Directions: Please do all of your work in this exam booklet and make sure that you cross out any work that we should ignore when we grade. **Books and extra papers are not permitted.** If you have a question about a problem, please ask. Remember: answers that are written logically and clearly will receive higher scores. There are 8 questions on 12 pages (not counting this cover page). Please make sure that you have all 12 pages of questions.

Do not write in the following box:

PROBLEM	POSSIBLE	SCORE
1	12	
2	12	
3	12	
4	12	
5	12	
6	12	
7	12	
8	16	
TOTAL	100	

1. (12 points) Calculate:

   (a) the gradient of the function $f(x,y) = e^{xy}\sin 2x$ at the point $(\pi/6, 0)$.

   (b) the projection $\mathrm{proj}_{\mathbf{b}}\mathbf{a}$ of the vector $\mathbf{a} = \mathbf{i} + 4\mathbf{j} - \mathbf{k}$ in the direction of $\mathbf{b} = 2\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$.

   (c) the curl of the vector field $\mathbf{F}(x,y,z) = x^3yz^2\mathbf{i} + 3x^3yz^2\mathbf{j} + 2x^2yz^2\mathbf{k}$.

2. (12 points) Which of the following five lines are parallel? Which are equal? **In order to receive any credit for your answer, you must provide brief justifications for your assertions.**

$$\ell_1 : x = 1 + t, \quad y = t, \quad z = 2 - 5t$$

$$\ell_2 : x + 1 = y - 2 = 1 - z$$

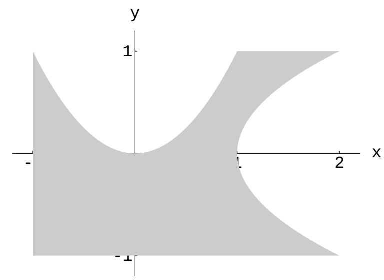
$$\ell_3 : x = 1 + t, \quad y = 4 + t, \quad z = 1 - t$$

$$\ell_4 : \mathbf{r}(t) = (2 + 2t)\mathbf{i} + (1 + 2t)\mathbf{j} - (3 + 10t)\mathbf{k}$$

$$\ell_5 : x = 2 + t, \quad y = 3 + t, \quad z = 2 + t$$

3. (12 points) Consider the following region R in the xy-plane. It is bounded by the curves $y = x^2$ and $x = y^2 + 1$ and the lines $y = -1$, $y = 1$, and $x = -1$. Calculate

$$\iint_R 6xy^2 \, dA.$$



4. (12 points) Consider the curves

$$\mathbf{r}_1(t) = e^t\mathbf{i} + t\mathbf{j} + (t-1)\mathbf{k} \qquad \text{and} \qquad \mathbf{r}_2(t) = t\mathbf{i} + (t-1)\mathbf{j} - t^2\mathbf{k}.$$

Determine their point(s) of intersection and the angle(s) at which they intersect.

5. (12 points) Find the point(s) on the cone $z^2 = x^2 + y^2$ that are closest to the point $(3, 1, 0)$.

6. (12 points) Find the volume of the solid region R lying inside the sphere $x^2+y^2+z^2=9$ and outside the cone $z^2=x^2+y^2$. To be precise,

$$R=\left\{(x,y,z)\,|\,x^2+y^2+z^2\leq 9 \text{ and } z^2\leq x^2+y^2\right\}.$$

7. (12 points) **Note that there is a second part to this problem on the next page.**

   (a) Fix a radius R. Using the surface area formula we discussed in this course, derive the surface area of a hemisphere of the form

$$x^2 + y^2 + z^2 = R^2 \quad \text{with} \quad z \geq 0.$$

Problem 7 (continued):

(b) With the aid of your result in part (a), calculate the x-coordinate $\bar{x}$ of the center of mass of that portion of the sphere $x^2+y^2+z^2=R^2$ that lies in the first octant (x, y, and z all positive). You can use the integral

$$\int \frac{u^2}{\sqrt{a^2-u^2}}\,du = -\frac{u}{2}\sqrt{a^2-u^2}+\frac{a^2}{2}\sin^{-1}\frac{u}{a}+C$$

without justification. Also, recall that $\bar{x}$ is the surface integral of x over the surface divided by the surface area of the surface.

8. (16 points) **Note: This problem has multiple parts on pages 9–12**

(a) Compute and classify the critical points of the function $f_1(x,y) = x^2 - 2x + y^2 - 2y$.

(b) Compute and classify the critical points of the function $f_2(x,y) = 3x - x^3 + y^2$.

Problem 8 (continued):

(c) Compute and classify the critical points of the function $f_3(x,y) = x^3 - 3x + y^2$.

(d) Compute and classify the critical points of the function $f_4(x,y) = xy - x - y$.

Problem 8 (continued):

(e) Compute and classify the critical points of the function $f_5(x,y) = x^4 + y^4 - 4xy$.

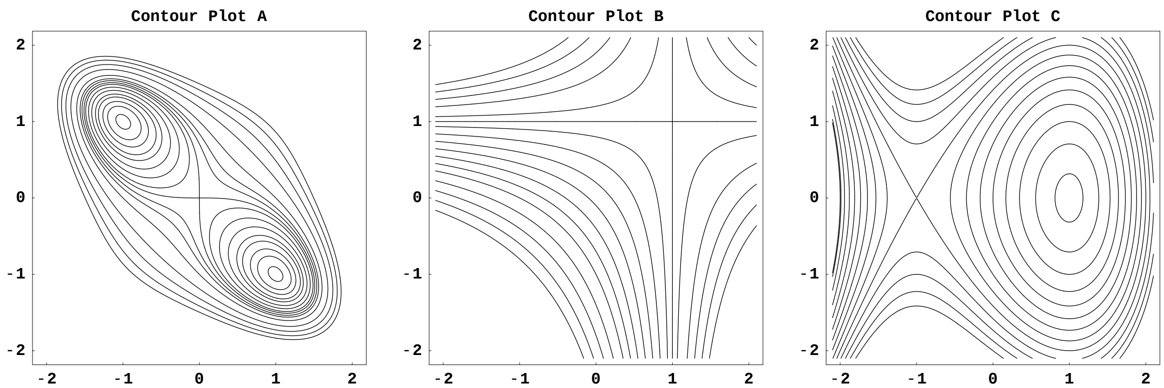
(f) Compute and classify the critical points of the function $f_6(x,y) = x^4 + y^4 + 4xy$.

Problem 8 (continued):

(g) The six functions considered in parts (a)–(f) are:

1. $f_1(x,y) = x^2 - 2x + y^2 - 2y$
2. $f_2(x,y) = 3x - x^3 + y^2$
3. $f_3(x,y) = x^3 - 3x + y^2$
4. $f_4(x,y) = xy - x - y$
5. $f_5(x,y) = x^4 + y^4 - 4xy$
6. $f_6(x,y) = x^4 + y^4 + 4xy$

Here are three contour plots:



Match each contour plot with its corresponding function $f(x,y)$ from the choices above. Using your results from parts (a)–(f), provide a brief justification for your selection. **You will not receive any credit for your answer unless you provide a valid justification.**

A. The function for contour plot A is ____. My reason for choosing this answer is:

B. The function for contour plot B is ____. My reason for choosing this answer is:

C. The function for contour plot C is ____. My reason for choosing this answer is: