MA 225 Sample Examinations

Here are the four examinations that were given in my MA 225 class in the Fall of 2007. You should use them to get an idea of the format of a typical test and to see the types of questions I ask. *You should not assume that the test questions this semester will be on the same topics.* In fact, you are always responsible for *all* of the material that we cover in class as well as *all* of the designated material from your text. The best way to study for my exams is to be sure that you are very comfortable with the homework assignments and the examples that I present in class. My tests often vary in difficulty (as you can see here), and your grade for the examination will be determined by a curve that will be announced in class after the examination is graded.

MA 225 Exam 1A October 3, 2007

Name: ______________________________ Last five digits of ID Number: __________

Discussion Section (circle yours): M 12–1 M 1–2 M 2–3 T 2:30–3:30 T 3:30–4:30

Directions: Please do all of your work in this exam booklet and make sure that you cross out any work that we should ignore when we grade. **Books and extra papers are not permitted.** If you have a question about a problem, please ask. Remember: answers that are written logically and clearly will receive higher scores. There are 5 questions on 5 pages (not counting this cover page). Please make sure that you have all 5 pages of questions.

Do not write in the following box:

PROBLEM	POSSIBLE	SCORE
Name, disc sec	3	
1	21	
2	18	
3	18	
4	18	
5	22	
TOTAL	100	

1. (21 points) Calculate:

   (a) An equation for the plane that contains the point $(4,1,-2)$ and is parallel to the plane $3x-y+z=2$.

   (b) Symmetric equations for the line that contains the two points $(1,2,0)$ and $(4,-2,1)$.

   (c) All unit vectors in the plane that are perpendicular to the vector $\mathbf{v}=\mathbf{i}-4\mathbf{j}$.

2. (18 points) Consider the two planes

$$x + y - z = 2 \qquad \text{and} \qquad 3x - 4y + 5z = 6.$$

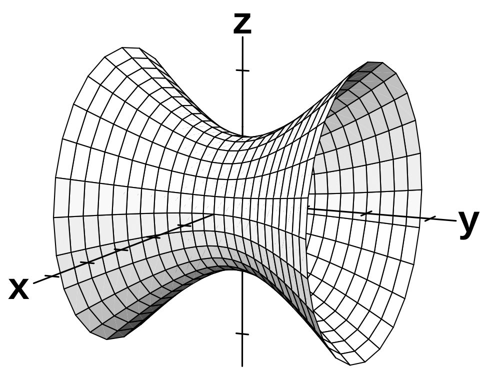
(a) Give a one-sentence justification of why they intersect.

(b) Find a vector equation for their line of intersection.

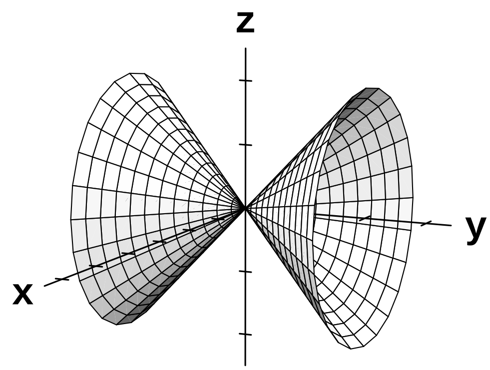
(c) Find the angle between these two planes.

3. (18 points) Here are two surfaces in space:

Surface A



Surface B



Here are six equations of surfaces:

1. $x^2 - y^2 + z^2 = 1$
2. $x - y + z = 1$
3. $x^2 - y^2 + z^2 = 0$
4. $x^2 + y^2 - z^2 = -1$
5. $x^2 - y^2 + z^2 = -1$
6. $x^2 - y^2 - z = 0$

For each surface, pick the equation that describes it. Provide a brief justification for your choice. **You will not receive any credit unless you provide a valid justification.**

(a) The equation for surface A is ____. My reason for choosing this answer is:

(b) The equation for surface B is ____. My reason for choosing this answer is:

4. (18 points) Find the area of the region that is enclosed by one loop of the polar curve $r = 2\sin 4\theta$. (The half-angle formula for sine is $\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$.)

5. (22 points) Consider the curve generated by the vector-valued function

$$\mathbf{r}(t) = t^2\,\mathbf{i} + (t^3 - 5t + 1)\,\mathbf{j} + t^2\,\mathbf{k}.$$

(a) Calculate the derivative $\mathbf{r}'(t)$.

(b) Calculate an equation in vector form for the line tangent to the curve at the point $(1, 5, 1)$.

(c) This curve intersects itself. Determine the point of self intersection and the angle of intersection.

MA 225	Exam 2A	October 31, 2007

Name: ______________________ Last five digits of ID Number: __________

Discussion Section (circle yours): M 12–1 M 1–2 M 2–3 T 2:30–3:30 T 3:30–4:30

Directions: Please do all of your work in this exam booklet and make sure that you cross out any work that we should ignore when we grade. **Books and extra papers are not permitted.** If you have a question about a problem, please ask. Remember: answers that are written logically and clearly will receive higher scores. There are 5 questions on 5 pages (not counting this cover page). Please make sure that you have all 5 pages of questions.

Do not write in the following box:

PROBLEM	POSSIBLE	SCORE
Name, disc sec	3	
1	21	
2	18	
3	18	
4	20	
5	20	
TOTAL	100	

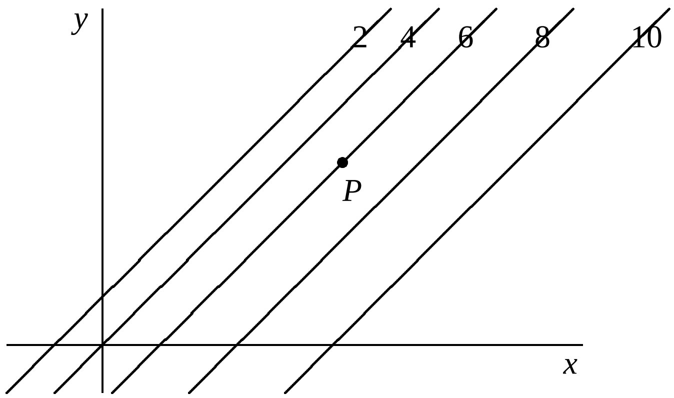
1. (21 points) Let $f(x,y) = e^{2xy}\sin 3x$. Calculate:

   (a) the partial derivative $\dfrac{\partial f}{\partial x}$

   (b) the gradient $\nabla f\left(\dfrac{\pi}{18}, 0\right)$

   (c) A vector $\mathbf{T}$ that is tangent to the level set of f at the point $\left(\dfrac{\pi}{18}, 0\right)$.

2. (18 points) The following curves are level sets of a function $f(x,y)$ corresponding to levels 2, 4, 6, 8, and 10.



For each derivative at the point P shown, indicate if it is most likely positive or negative, and provide a brief justification for your answer. **You will not receive any credit unless you provide a valid justification.**

(a) Is $\dfrac{\partial f}{\partial x}$ positive or negative at P? Why?

(b) Is $\dfrac{\partial f}{\partial y}$ positive or negative at P? Why?

(c) Is $\dfrac{\partial^2 f}{\partial x^2}$ positive or negative at P? Why?

3. (18 points) The length l, width w, and height h of a rectangular box are changing with time t.

(a) State the version of the Chain Rule that specifies how the volume v of the box changes with time.

(b) At a given instant t_0, $l = 2$ m, $w = 1$ m, and $h = 3$ m. Also, at the same instant, l is increasing at a rate of 2 m/s, w is increasing at a rate of 3 m/s, and h is decreasing at a rate of 2 m/s. At what rate is the volume of the box changing at time t_0?

4. (20 points) Find the average value of the function $f(x,y) = 4xye^{x^2y}$ over the rectangle

$$R = \{(x,y)\,|\,0 \le x \le 2,\ 0 \le y \le 3\}.$$

5. (20 points) Find the point(s) on the surface $x^2 + y^2 - z^2 = 2$ that are closest to the point $(2, 3, 0)$.

MA 225	Exam 3A	December 5, 2007

Name: ________________________________ Last five digits of ID Number: __________

Discussion Section (circle yours): M 12–1 M 1–2 M 2–3 T 2:30–3:30 T 3:30–4:30

Directions: Please do all of your work in this exam booklet and make sure that you cross out any work that we should ignore when we grade. **Books and extra papers are not permitted.** If you have a question about a problem, please ask. Remember: answers that are written logically and clearly will receive higher scores. There are 5 questions on 5 pages (not counting this cover page). Please make sure that you have all 5 pages of questions.

Do not write in the following box:

PROBLEM	POSSIBLE	SCORE
Name, disc sec	3	
1	19	
2	20	
3	18	
4	20	
5	20	
TOTAL	100	

1. (19 points) Let C be the positively-oriented boundary of the region enclosed by the parabolas $y = x^2$ and $x = y^2$. Use Green's Theorem to evaluate the line integral

$$\oint_C (y + e^{\sqrt{2x}})\,dx + (3x^2 + \cos y^2)\,dy.$$

2. (20 points) Compute the volume of the solid that lies under the plane $z = 4x$ and above the region in the xy-plane bounded by $y = 3x$ and $y = x^2 - 2x$.

3. (18 points) Evaluate the triple integral

$$\iiint\limits_R \sqrt{x^2+y^2+z^2}\, dV$$

where R is that part of the solid region $x^2+y^2+z^2 \leq 2$ that lies in the first octant ($x, y, z \geq 0$).

4. (20 points) The base of a fence is a semicircle with a radius of 10 feet. Using a coordinate system where this base is described by the curve $x^2 + y^2 = 100$, with $y \geq 0$, the height of the fence at the point (x, y) is $4 + x/5 + y/10$ feet. Calculate the surface area of the fence.

5. (20 points) Find the surface area of the portion of the circular paraboloid $z = 16 - x^2 - y^2$ that lies between the planes $z = 4$ and $z = 12$.

MA 225 | Final A | December 17, 2007

Name: ________________________________ Last five digits of ID Number: ____________

Directions: Please do all of your work in this exam booklet and make sure that you cross out any work that we should ignore when we grade. **Books and extra papers are not permitted.** If you have a question about a problem, please ask. Remember: answers that are written logically and clearly will receive higher scores. There are 9 questions on 10 pages (not counting this cover page). Please make sure that you have all 10 pages of questions.

Do not write in the following box:

PROBLEM	POSSIBLE	SCORE
1	9	
2	9	
3	8	
4	12	
5	12	
6	12	
7	12	
8	12	
9	14	
TOTAL	100	

1. (9 points) Calculate:

   (a) The angle between the vectors $\mathbf{A} = \mathbf{i} + \sqrt{6}\,\mathbf{j} + \mathbf{k}$ and $\mathbf{B} = -\mathbf{i} + \sqrt{6}\,\mathbf{j} - \mathbf{k}$.

   (b) An equation for the plane that contains the point $(3, 2, 0)$ and is perpendicular to the line
   $$x - 1 = \frac{y-1}{2} = \frac{z-1}{3}.$$

   (c) A normal vector for the plane that contains the two lines
   $$x - 1 = \frac{y-1}{3} = \frac{z-1}{2} \quad \text{and} \quad x = y = z.$$

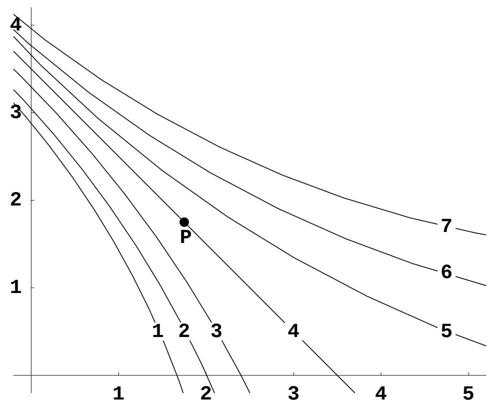
2. (9 points) Calculate:

   (a) Suppose that $\nabla f(2,1) = 4\mathbf{i} + 3\mathbf{j}$ for some function $f(x,y)$. Compute the directional derivative of f in the direction of the vector $\mathbf{v} = 2\mathbf{i} - \mathbf{j}$.

   (b) The divergence of the vector field $\mathbf{F}(x,y,z) = xy^2z^3\mathbf{i} + 2x^2yz^3\mathbf{j} + 3x^2y^2z^2\mathbf{k}$.

   (c) An equation of the plane tangent to $z = x^2 + xy + y^3$ at the point $(1,2,11)$.

3. (8 points) **Note: This problem continues on the next page.** The following curves are level sets of a function $f(x,y)$ corresponding to levels 1, 2, 3, 4, 5, 6, and 7.

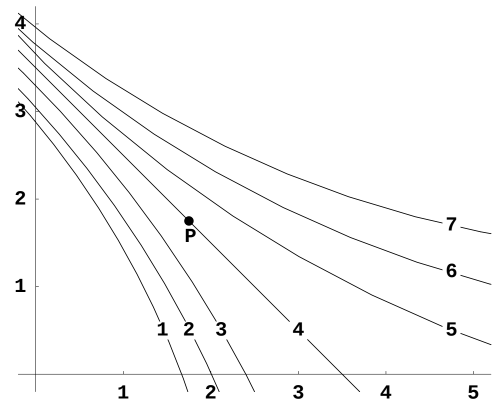


For each derivative at the point P, indicate if it is most likely positive or negative, and provide a one- or two-sentence justification for your answer. **You will not receive any credit unless you provide a valid justification.**

(a) Is $\dfrac{\partial f}{\partial x}$ positive or negative at P? Why?

(b) Is $\dfrac{\partial f}{\partial y}$ positive or negative at P? Why?

Problem 3 (continued): The contours are repeated here for your convenience.



(c) Is $\dfrac{\partial^2 f}{\partial x^2}$ positive or negative at P? Why?

(d) Is $\dfrac{\partial^2 f}{\partial y \partial x}$ positive or negative at P? Why?

4. (12 points) Calculate the line integral

$$\int_C (x+y)\,dx + 3x\,dy + xy\,dz,$$

where the curve C consists of a line segment that goes from $(2,0,2)$ to $(3,3,2)$ followed by a line segment that goes from $(3,3,2)$ to $(3,5,3)$.

5. (12 points) Calculate the arc length of the curve swept out by the vector-valued function

$$\mathbf{r}(t) = 6t\,\mathbf{i} + 3t^2\,\mathbf{j} + t^3\,\mathbf{k}$$

from the point $(-12, 12, -8)$ to the point $(6, 3, 1)$.

6. (12 points) Show that the line integral

$$\int_C (1 - ye^{-x})\,dx + (e^{-x} + y)\,dy$$

is independent of path. What is its value over any path C from $(0,1)$ to $(1,3)$?

7. (12 points) Evaluate the triple integral

$$\iiint_E 2z\,dV,$$

where E is the solid that lies inside the cylinder $x^2+y^2=3$, above the plane $z=0$, and below the cone $z^2=4x^2+4y^2$.

8. (12 points) Let $\mathbf{F}(x,y,z) = x\mathbf{i} + y\mathbf{j} + z^4\mathbf{k}$ and S be the part of the cone $z = \sqrt{x^2+y^2}$ with $z \leq 2$. Calculate the flux

$$\iint_S \mathbf{F}\cdot\mathbf{n}\, dS,$$

using the orientation of S that points downward.

9. (14 points) Evaluate the triple integral

$$\iiint_T 2yz\,dV,$$

where T is the solid tetrahedron with vertices $(0,0,0)$, $(0,1,0)$, $(2,1,0)$, and $(0,1,2)$.