MA 226 Spring 2016
Project 1

Due: At the **start of class** on Friday, March 18.

Group project: This is a group project with each group having either three or four members. Once a group begins work on the project, its membership cannot change. Consequently establishing your group must be your first step in this project. Each group will submit one report, and all members of the group will receive the same grade for this project.

For this project you will determine your groups. However, if you would like my help organizing a group, please send me email (`paul@bu.edu`) immediately. I plan to send out lists of suggested partners during or before the weekend of February 27–28.

Your report: Your report should be no longer than four typewritten pages, and it should address all of the questions in the project description. Your report should include a cover page followed by an academic conduct signature page. A blank signature page is provided at the end of this document. Neither of these two pages count as part of the four-page limit.

You may provide as many illustrations from the computer as you wish, but the relevance of each illustration to your report must be evident. Illustrations are part of the four-page limit. Please insert your illustrations at appropriate places in your report rather than attaching them to the end of the report. Examples of good reports done here at BU in previous semesters are available for inspection during my Wednesday and Friday office hours and during discussion section.

Project description: Autonomous second-order differential equations are studied numerically by reducing them to first-order systems with two dependent variables. In this project you will use the computer to analyze three somewhat related second-order equations. In particular, you will analyze phase planes and $y(t)$- and $v(t)$-graphs to describe the long-term behavior of the solutions.

In Sections 2.1 and 2.3, we discuss the most classic of all second-order equations

$$m\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ky = 0.$$

It is an example of a second-order, homogeneous, linear equation with constant coefficients. In the text we explain how this equation is used to model the motion of a spring. The force due to the spring is assumed to obey Hooke's law (the force is proportional to the amount the spring is compressed or stretched). The force due to damping is assumed to be proportional to the velocity. In your report you should describe the motion of the spring assuming certain values of the parameters m, b, and k. (A table of values of the parameters is given below. See the section entitled "Parameter values" for directions on which values of the parameters you should use.)

Your report should discuss the following:

1. *Undamped harmonic oscillator:* The first equation that you should study is the harmonic oscillator with no damping; that is, $b = 0$. Examine solutions using both their graphs and the phase plane. Are the solutions periodic? If so, what does the period seem to be? Describe the behavior of three different solutions that have **especially different initial conditions** (in terms of their physical interpretation) and be specific about their physical interpretation. (Analytic methods to answer these questions are discussed in Chapter 3. For now, work numerically.)

2. *Harmonic oscillator with damping:* Repeat Part 1 using the equation

$$m\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ky = 0,$$

   where b is the nonzero value given on page 3.

3. *Harmonic oscillator with nonlinear damping:* Repeat Part 1 using the equation

$$m\frac{d^2y}{dt^2} + b\left|\frac{dy}{dt}\right|\frac{dy}{dt} + ky = 0$$

   in place of the usual harmonic oscillator equation. (Note that even with the same value of the parameter b, the drag forces in this equation and the equation in Part 2 have the same magnitude only for velocity ± 1. Also, note that the sign of the term

$$\left|\frac{dy}{dt}\right|\frac{dy}{dt}$$

   is the same as the sign of dy/dt. Consequently this damping force is always directed opposite the direction of motion.) The difference between this equation and that in Part 2 is the size of the damping for small and large velocities.

4. *Nonlinear second-order equation:* Finally, consider a somewhat related second-order equation where the damping coefficient b is replaced by the factor $(y^2 - \alpha)$; that is,

$$m\frac{d^2y}{dt^2} + (y^2 - \alpha)\frac{dy}{dt} + ky = 0.$$

   Is it reasonable to interpret this factor as some type of damping? Provide a complete description of the long-term behavior of the solutions. Are the solutions periodic? If so, what does the period seem to be? Explain why this equation is not a good model for something like a mass-spring system. Give an example of some other type of physical or biological phenomenon that could be modeled by this equation.

Your report (again): Address the questions in each item above in the form of a short essay. Be particularly sure to describe the behavior of the solution and the corresponding behavior of the mass-spring system. You may use the phase planes and graphs of $y(t)$ to illustrate the points you make in your essay. (However, please remember that, although one good illustration may be worth 1000 words, 1000 illustrations are usually worth nothing.)

Your parameter values: Determine the values of the parameters m, k, b, and α as follows:

1. Average the last digits of your BU ID numbers.

2. Round this number to the nearest integer (round up for .5).

3. Use the result to pick your parameter values from the table below.

For example, if you are working in a group of four and your ID numbers end in 3, 1, 4, and 1, then the average is 2.25, and you use the parameter values that correspond to a rounded average of 2.

Rounded Average	m	k	b	α
0	5	4	4	2
1	2	5	2	3
2	3	5	3	3
3	5	5	4	3
4	2	6	3	5
5	3	6	3	5
6	5	6	3	5
7	5	4	4	2
8	5	5	4	2
9	5	6	4	2
10	5	4	4	2

Asking questions: Questions by email about the project are not allowed. Questions about the project will be answered using the discussion forum for our MA226 course on edge.edx.org. I will check the forum for questions once each day. The last time that I will check the forum will be 11 am on Thursday, March 17. Do not wait until the last minute to ask questions.

Academic Conduct: Your work and conduct in this course are governed by the Boston University Academic Conduct Code. A copy of this code is available at

http://www.bu.edu/academics/files/2011/08/AcademicConductCode.pdf

This code is designed to promote high standards of academic honesty and integrity as well as fairness. It is your responsibility to know and follow the provisions of the code. **In particular, all work that you submit in this course must be your original work.** For example, you can only discuss your project with other members of your group, with Professor Blanchard or Eric. Moreover, the computations that you do for your report as well as the text of your report must be original to your group. All group members are responsible for all aspects of the report. If you have a question about any aspect of academic conduct, please ask.

We understand that this project report is governed by the Boston University Academic Conduct Code. In particular, this code requires that the work submitted here is our original work and that we performed the computational work included in this report. Moreover, we have not discussed our results with anyone either than Professor Blanchard or Eric Chang.

Name (print)

Signature

BU ID number

Date

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