Forced Cantilever Beam Project
Project Description

We begin the project by considering the forced harmonic oscillator

$$\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = \cos \omega t.$$

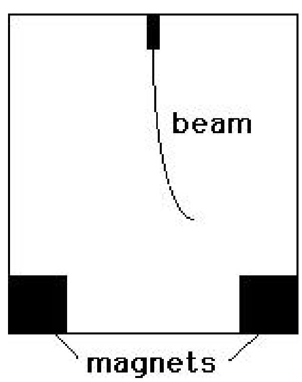
Here b is the damping coefficient, k is the spring constant, and ω determines the frequency of the forcing.

1. (An undamped, forced linear equation) First consider the equation in the case where $b = 0$. Using the Method of the Lucky Guess, determine a particular solution to the equation. Then using the formulas for the solutions, estimate the amplitudes of the solutions for frequencies ω in the interval $0 \leq \omega \leq 2$. How do the amplitudes in the nonresonant case relate to the amplitudes in the resonant case?

2. (Damped, forced linear equations) Assume that the forcing is present along with some damping ($b > 0$). How long does it take any given solution to get close to the steady-state solution? Using `HPGSystemSolver` and/or the formulas for the solutions, estimate the maximum amplitudes of the solutions for frequencies ω in the interval $0 \leq \omega \leq 2$ for various values of b. Graph the maximum amplitude as a function of frequency for different values of b, all on the same set of axes. What happens to these graphs as $b \to 0$?

Now we return to the cantilever beam that we studied in the first project. We will assume that the magnets are strong. Consequently, the differential equation for the cantilever beam is

$$\frac{d^2x}{dt^2} + b\frac{dx}{dt} - kx + x^3 = A \cos \omega t.$$



The $A \cos \omega t$ term is introduced to model a periodic external forcing in the system which can be considered to be the effect of someone periodically shaking the box.

3. (strong magnets with no forcing) Let $A = 0$. Repeat your analysis in part 4 of Project 1. Note that you now have the mathematical tools to compute the bifurcation value of b directly. Sketch those computations.

4. (strong magnets with forcing) Now we turn on the forcing by making $A > 0$. Use $\omega = 2$ and $b = 0.2$ throughout this part of the project. Your goal is to give a physical interpretation of the "steady-state" solutions for this system for various values of A.

   Finding stable periodic solutions of the system is not easy because there is no standard technique to do so. I recommend going back and forth between `HPGSystemSolver` and a new program called `CantileverPMap`. (`CantileverPMap` will be distributed via the course web page. It will be an implementation of the so-called Poincaré map for this periodically forced equation. For more details about Poincaré maps, see Section 5.6 in your textbook.) When you use `HPGSystemSolver`, use a small step size to control numerical error. I recommend $\Delta t = 0.001$.

   Start with a very small value for A such as $A = 0.1$ (or smaller if necessary) and use what you know about the $A = 0$ case. After you find periodic solutions in this case, increase A and use what you learned previously to find new periodic solutions. Repeat this process many times.

   Try to find as many "different" periodic solutions as you can for various values of A. For each such solution, specify its initial condition and period, and give a physical description of the motion of the beam as the solution evolves over time. (It is even possible to find many different periodic solutions for the same value of A if you are careful or somewhat lucky.)