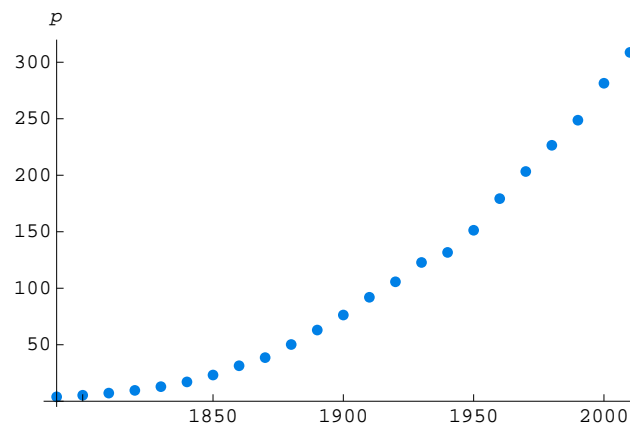


Modeling the U.S. Population:

The data graphed as a function of time



Steps in Model Building

1. State underlying assumptions.
2. Identify the relevant variables and parameters.
3. Use the assumptions in Step #1 to formulate equations relating the variables in Step #2.

First Model: Malthusian Model

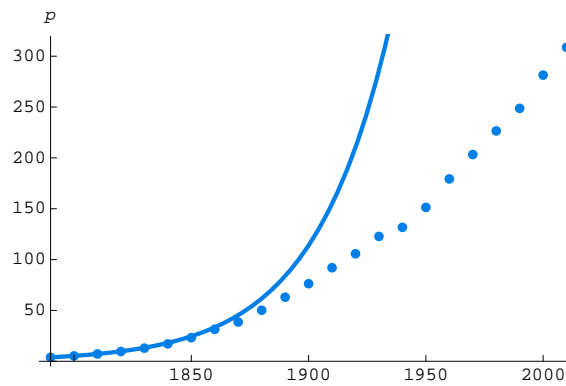
Assumption: Growth rate of the population is proportional to the population.

Variables:

Malthusian model is

Analytic technique:

Here's the graph of $p(t)$ superimposed on the data:



Second Model: Logistic Model

Assumptions:

1. If the population is small, its growth rate is proportional to the size of the population.
2. As the population increases, its **relative growth rate** decreases.

What is a relative growth rate?

A Qualitative Analysis of the Logistic Model

We now have

$$\frac{dp}{dt} = kp \left(1 - \frac{p}{N} \right).$$

Can we determine the long-term behavior of solutions without computing the solutions first?