More general comments

At the end of last class, I started to make some general comments about first-order differential equations, and I want to continue with those comments now.

1. What does it mean to solve the initial-value problem

$$\frac{dy}{dt} = f(t, y), \qquad y(t_0) = y_0?$$

   A solution to the initial-value problem is a differentiable function $y(t)$ defined on some interval $a < t_0 < b$ containing t_0 such that

   (a) $y(t_0) = y_0$ and

   (b) $\frac{dy}{dt} = f(t, y(t))$ for all t in the interval $a < t < b$.

2. Be careful about notation: The distinction between the independent and the dependent variables is important.

   Example 1. $\frac{dy}{dt} = kt$

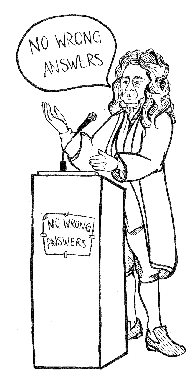
   The solutions to this equation are $y(t) = k\frac{t^2}{2} + c$, where c is an arbitrary constant.

   Example 2. $\frac{dy}{dt} = ky$

   The solutions to this equation are $y(t) = y_0 e^{kt}$, where y_0 is an arbitrary constant.

3. What does the term **general solution** mean?

4. You should never get a wrong answer in this course:

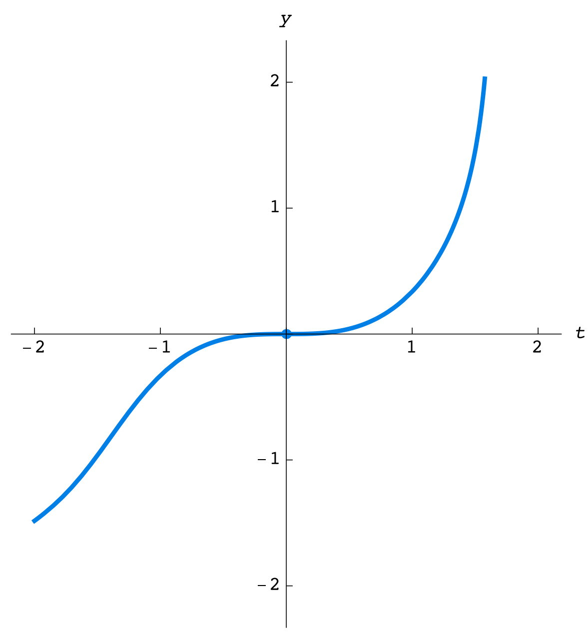


5. Even relatively simple looking differential equations can have solutions that cannot be expressed in terms of functions that we already know and love.

   Consider the initial-value problem

$$\frac{dy}{dt} = y^3 + t^2, \qquad y(0) = 0.$$

   Here is the graph of the solution as generated by `HPGSolver`.



Our general approach in this course:

We will study differential equations

1. using the theory and
2. various techniques:
   (a) analytic techniques
   (b) geometric/qualitative techniques, and
   (c) numerical techniques.

Separable Differential Equations (an analytic technique)

First let's recall the method of substitution for calculating integrals (really antiderivatives):

A differential equation

$$\frac{dy}{dt} = f(t, y)$$

is **separable** if it can be written in the form

$$\frac{dy}{dt} =$$

Two Examples:

1. $\frac{dy}{dt} = -2ty^2$

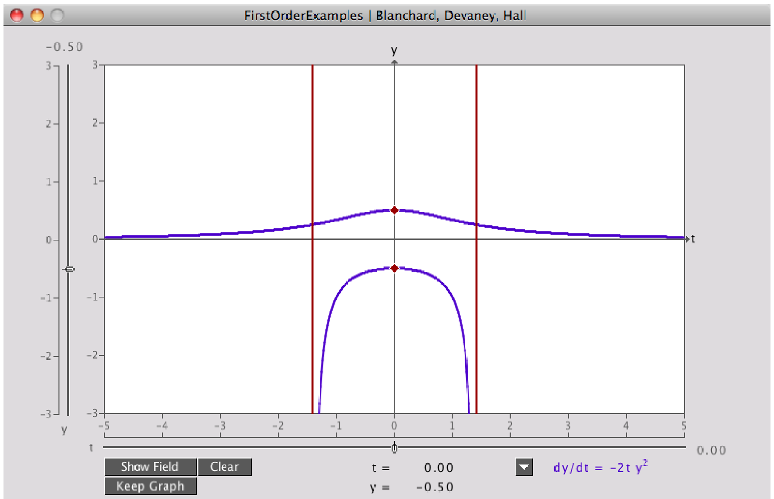
2. $\frac{dy}{dt} = y^3 + t^2$

Let's go back to the first example

Example. $\frac{dy}{dt} = -2ty^2$

Let's solve two initial-value problems:

We turn to `FirstOrderExamples` to get a sense of the graphs of these solutions:

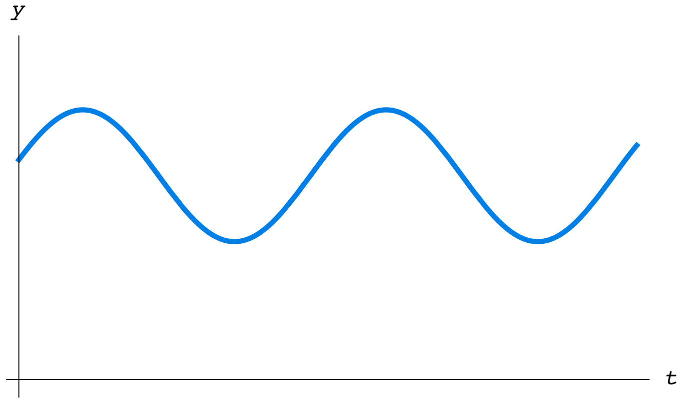


What's the general solution to $\frac{dy}{dt} = -2ty^2$? (Think before you answer.)

Slope fields

A **slope field** in the ty-plane is a picture of a first-order differential equation

$$\frac{dy}{dt} = f(t, y).$$



The graph of a solution must be everywhere tangent to the slope field.

Example. Once again consider the differential equation $\dfrac{dy}{dt} = -2ty^2$.