More on the method of the lucky guess

Method of the Lucky Guess:

1. Solve the associated homogeneous equation.
2. Guess one solution to the nonhomogeneous equation.

Example 2. $\dfrac{dy}{dt} = -y + 2\cos 4t$

1. General solution of the associated homogeneous equation: $y(t) = ke^{-t}$

2. Particular solution of the nonhomogeneous equation:

   We guessed $y_p(t) = \alpha \cos 4t + \beta \sin 4t$. Then

$$\frac{dy_p}{dt} + y = (\alpha + 4\beta)\cos 4t + (-4\alpha + \beta)\sin 4t$$

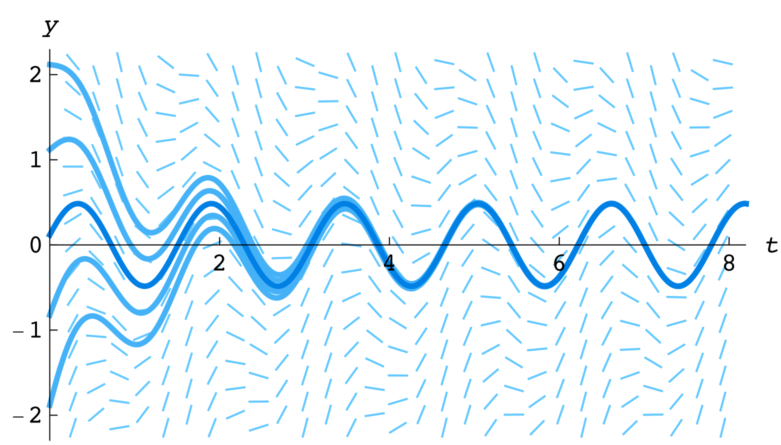
$$\stackrel{?}{=} 2\cos 4t.$$

   We want α and β such that

$$\begin{cases} \alpha + 4\beta = 2 \\ -4\alpha + \beta = 0. \end{cases}$$

   We get

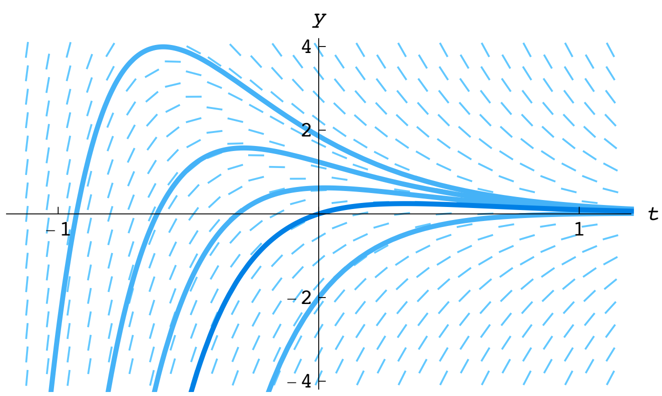
$$\alpha = \frac{2}{17} \quad \text{and} \quad \beta = \frac{8}{17}.$$



Example 3. $\dfrac{dy}{dt} = -3y + 2e^{-3t}$

1. General solution of the associated homogeneous equation:

2. Particular solution of the nonhomogeneous equation (trick question):



The method of integrating factors

For this method we rewrite the nonhomogeneous equation

$$\frac{dy}{dt} = a(t)y + b(t)$$

as

$$\frac{dy}{dt} + g(t)y = b(t).$$

In other words, $g(t) = -a(t)$. There is no mathematical significance to this step. It just avoids an annoying minus sign in one of the formulas.

Now for some magic:

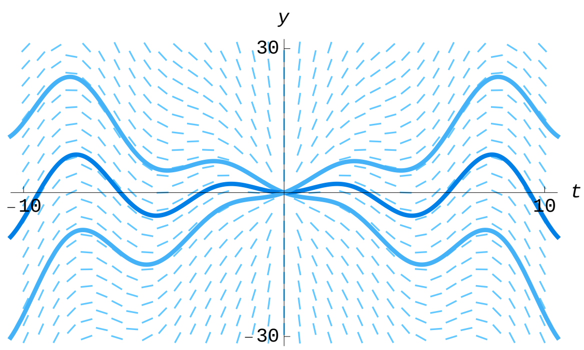
Summary: Given a linear differential equation of the form

$$\frac{dy}{dt} + g(t)y = b(t),$$

the integrating factor (magic function) is $\mu(t) = e^{\left(\int g(t)\,dt\right)}$.

Example. $\displaystyle \frac{dy}{dt} = \frac{y}{t} + t\cos t$

(see `FirstOrderExamples` in `DETools` and the example on the bottom of page 4 of the notes for February 5).



Recall one of the examples that we did at the start of class.

Example 3. $\dfrac{dy}{dt} = -3y + 2e^{-3t}$

Now we have two methods for solving nonhomogeneous linear equations. How do we decide which method to use?

Method of integrating factors:

1. In theory, it always works.
2. But you must calculate two integrals.

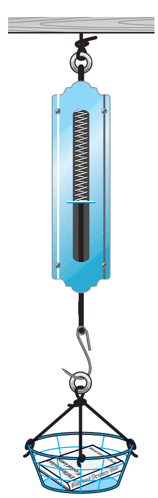
Method of the lucky guess:

1. Tends to be faster than the method of integrating factors—mostly algebra.
2. Note that the examples that we considered were all of the form

$$\frac{dy}{dt} = a(t)y + b(t)$$

   where $a(t)$ was a constant function ("constant-coefficient case"). What types of functions $b(t)$ did we consider?
3. But you need to be lucky.

Simple mass-spring system

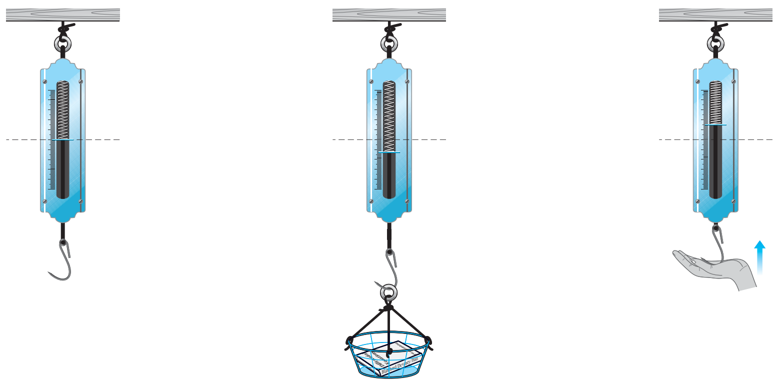


We model the motion of a mass-spring system using Newton's second law

$$F = ma$$

and Hooke's law. Hooke's law asserts that the restoring force of a spring is linearly proportional to its displacement from its rest position.

Hooke's law: the restoring force of a spring is linearly proportional to its displacement from its rest position.



Using Newton's second law $F = ma$ and Hooke's law, we get

MA 226 Exam Logistics

1. Bring pen/pencil and id. Closed book exam. No extra papers. No calculators, ipods, phones, etc.
2. Exam will start promptly at 9:30 and end at 10:50.
3. We will collect exams by moving up the aisles. You must pass in your exam when we arrive at your aisle. Please remain seated and quiet until we collect the exams from your aisle.
4. Five minute rule will be in effect: No one will be allowed to leave the exam between 10:45 and 10:50. Use those 5 minutes to check your work. (You should never get a wrong answer in this course.)
5. Seating will be assigned before the exam starts.
6. If you have a question, raise your hand. Stay seated.
7. Go to the bathroom before the exam starts.