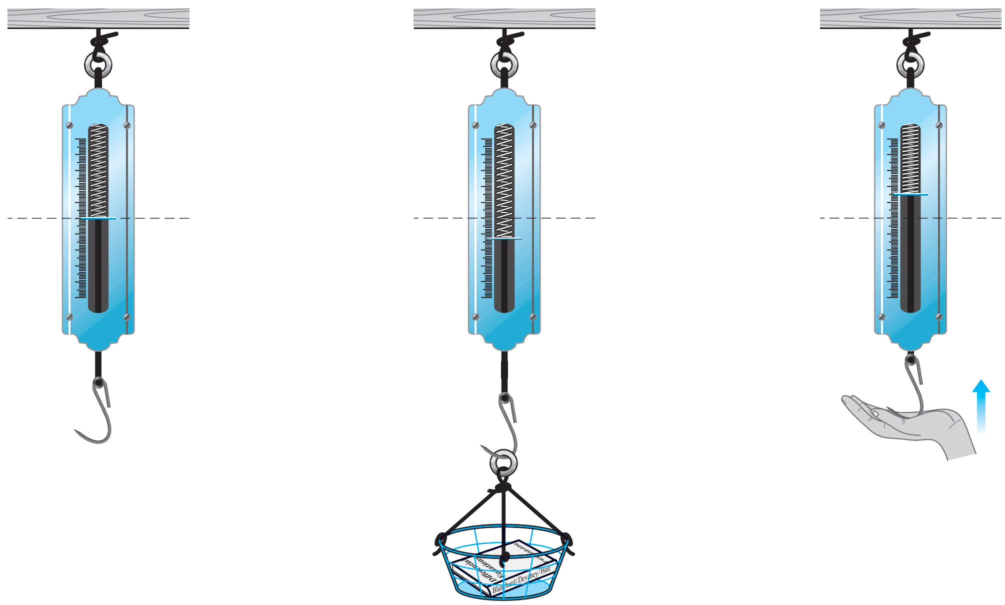
Simple mass-spring system

We model the motion of a mass-spring system using Newton's second law $F = ma$ and Hooke's law.

Hooke's law: the restoring force of a spring is linearly proportional to its displacement from its rest position.



Using Newton's second law $F = ma$ and Hooke's law, we get

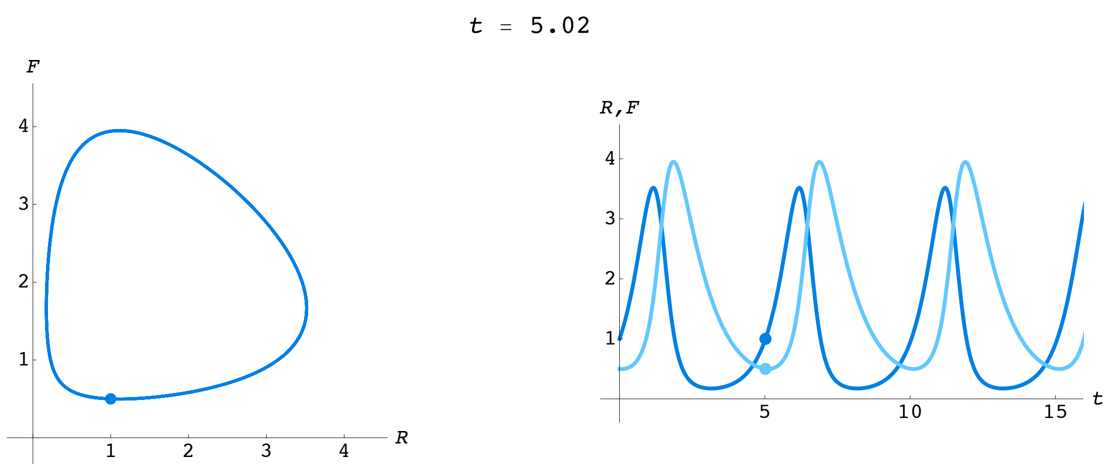
Predator-prey system

Example. Recall the predator-prey systems we discussed briefly at the start of the semester

$$\frac{dR}{dt} = aR - bRF$$
$$\frac{dF}{dt} = -cF + dRF.$$

Let's go through some terminology:

- initial condition:
- solution to an initial-value problem:



The solution shown above corresponds to the initial condition $(R_0, F_0) = (1, 0.5)$ with parameter values $a = 2$, $b = 1.2$, $c = 1$, and $d = 0.9$. `DETools` also has a tool called `PredatorPrey`.

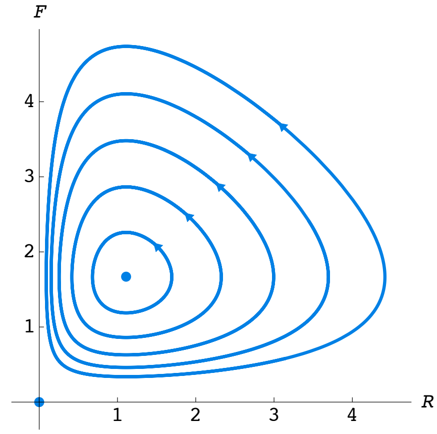
- component graphs:

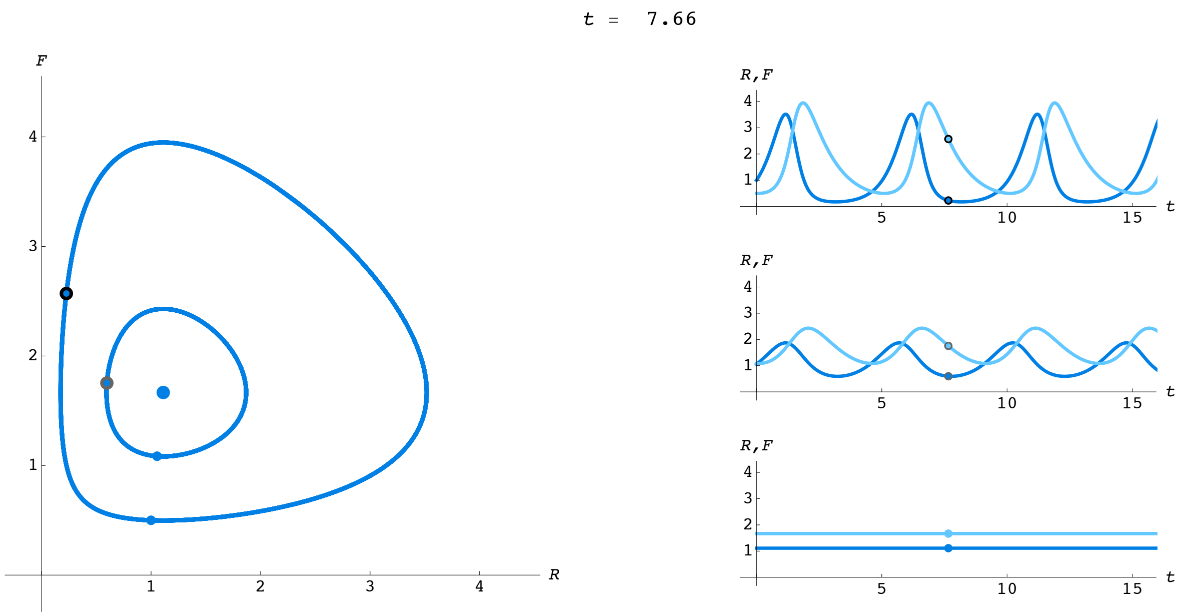
- phase plane:

- solution curve in the phase plane:

- equilibrium solutions:

- phase portrait:





One skill that you will learn is how to make a rough sketch of the component graphs from the solution curve. There is a tool on your CD called `DESketchPad` which will help you practice.

Reduction of order

Although the mass-spring system and the predator-prey system quite different, they have more in common than one might think. In particular, the mass-spring system can also be written as a first-order system by introducing the “new” variable v (which is just dy/dt). We get

In what ways is the mass-spring system similar to the predator-prey system?

An initial condition for the predator-prey system is a pair (R_0, F_0) of population values.

An initial condition for the mass-spring system is also a pair (y_0, v_0). The first number indicates the initial displacement and the second number indicates the initial velocity.

Since the mass-spring system can be expressed as a system, all of the terms that we discussed for the predator-prey system apply to the mass-spring system as well (equilibrium solutions, component graphs, phase plane, solution curve, phase portrait, ...).

One way that the two systems differ is by the fact that we can find formulas for the solutions of the mass-spring system but not for the predator-prey system.

For example, consider the special case of the mass-spring system where $k = m$. We get

$$\frac{d^2y}{dt^2} = -y,$$

and we can guess some solutions to this equation: