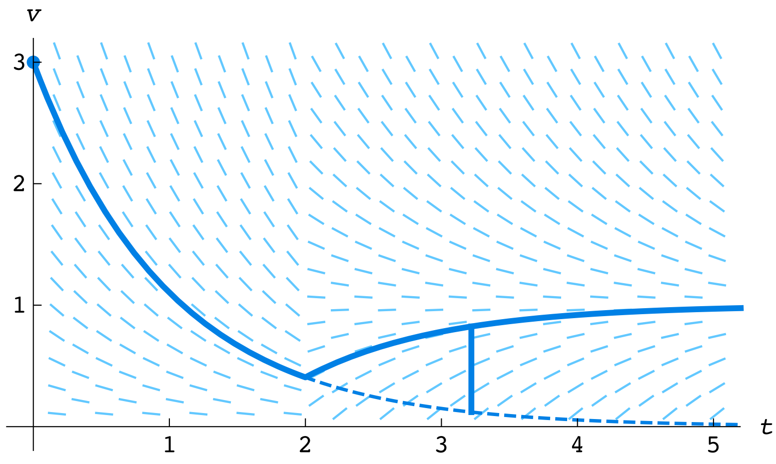
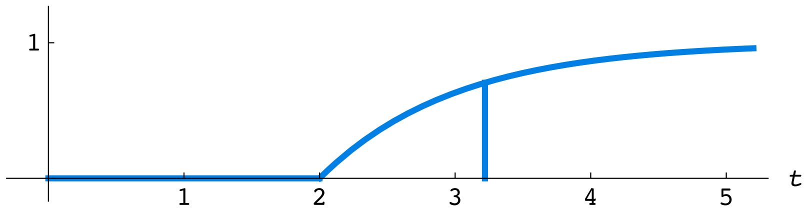
A comment about the solution to the initial-value problem from last class

Here are the graphs that I showed last class when we solved the initial-value problem

$$\frac{dv}{dt} + v = u_2(t), \quad v(0) = 3.$$



the solution $v(t) = 3e^{-t} + u_2(t)\left(1 - e^{-(t-2)}\right)$



the function $u_2(t)\left(1 - e^{-(t-2)}\right)$

More on the Laplace transform and second-order differential equations

Last class we derived three formulas that are relevant when the Laplace transform is applied to second-order linear equations. We obtained

$$\mathcal{L}\left[\frac{d^2y}{dt^2}\right] = s^2\mathcal{L}[y] - (y(0))s - y'(0),$$

and we used the fact that

$$\mathcal{L}[e^{i\omega t}] = \frac{1}{s - i\omega} = \frac{s + i\omega}{s^2 + \omega^2}$$

to derive

$$\mathcal{L}[\cos\omega t] = \frac{s}{s^2 + \omega^2} \qquad \text{and} \qquad \mathcal{L}[\sin\omega t] = \frac{\omega}{s^2 + \omega^2}.$$

Now that we know the transforms of sine and cosine, let's see how we use them.

Example. Compute

$$\mathcal{L}^{-1}\left[\frac{2s+1}{s^2+9}\right].$$

Now for a little practice with the third rule for transforms. Recall that

$$\mathcal{L}[u_a(t)f(t-a)] = e^{-as}\mathcal{L}[f].$$

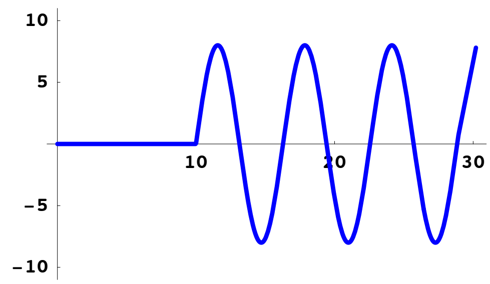
Example. Compute

$$\mathcal{L}^{-1}\left[\frac{8e^{-10s}}{(s^2+9)(s^2+1)}\right].$$

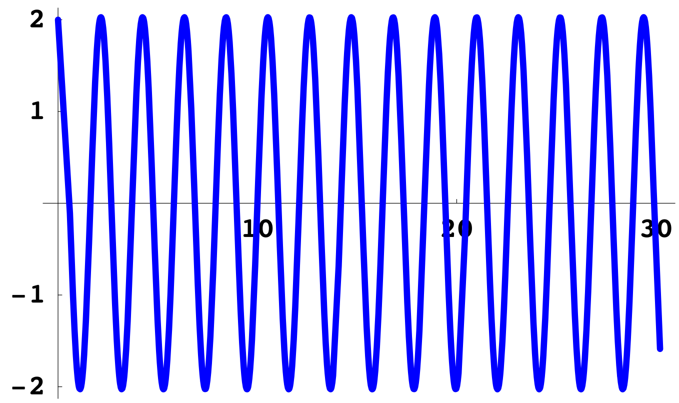
Now let's use what we have learned to solve the initial-value problem

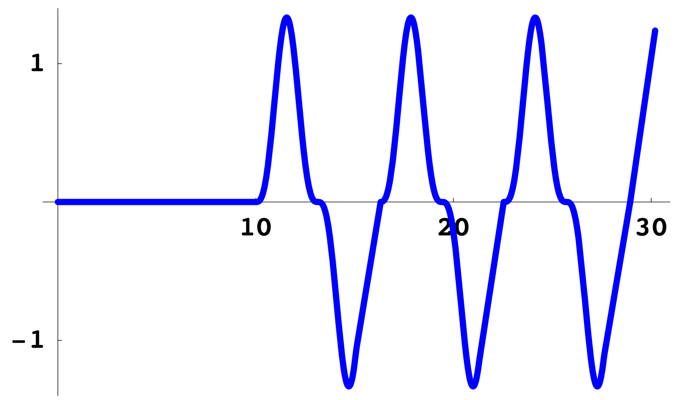
$$\frac{d^2y}{dt^2} + 9y = 8u_{10}(t)\sin(t-10), \quad y(0) = 2, \quad y'(0) = 1.$$

Here is the graph of the forcing function $8u_{10}(t)\sin(t-10)$:



Here are the graphs of the two functions that combine to give us the desired solution.





Here is the graph of the solution