Three examples to illustrate the geometry of complex eigenvalues:

**Example 1.** \( \frac{dY}{dt} = AY \) where

\[
A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}.
\]

The characteristic polynomial of \( A \) is \( \lambda^2 + 1 \), so the eigenvalues are \( \lambda = \pm i \). One eigenvector associated to the eigenvalue \( \lambda = i \) is

\[
Y_0 = \begin{pmatrix} i \\ 1 \end{pmatrix}.
\]
Example 2. $\frac{dY}{dt} = BY$ where

$$B = \begin{pmatrix} 2 & -2 \\ 4 & -2 \end{pmatrix}.$$

The characteristic polynomial of $B$ is $\lambda^2 + 4$, so the eigenvalues are $\lambda = \pm 2i$. One eigenvector associated to the eigenvalue $\lambda = 2i$ is

$$Y_0 = \begin{pmatrix} 1 + i \\ 2 \end{pmatrix}.$$
Example 3. $\frac{dY}{dt} = CY$ where

$$C = \begin{pmatrix} 1.9 & -2 \\ 4 & -2.1 \end{pmatrix}.$$  

The characteristic polynomial of $C$ is $\lambda^2 + 0.2\lambda + 4.01$, so the eigenvalues are $\lambda = -0.1 \pm 2i$. One eigenvector associated to the eigenvalue $\lambda = -0.1 + 2i$ is

$$Y_0 = \begin{pmatrix} 1 + i \\ 2 \end{pmatrix}.$$