Last class we discussed how the Extended Linearity Principle applies to second-order, nonhomogeneous, linear equations, and we solved one equation using a guessing technique.

Here's another example that looks similar but goes somewhat differently.

**Example 2.** Consider the equation

$$\frac{d^2y}{dt^2} - \frac{dy}{dt} - 2y = e^{-t}.$$

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A time saver: There's a calculation that we've already done twice before. It is also useful for guessing $y_p(t)$. Consider the function $y_p(t) = \alpha e^{\lambda t}$ and calculate

$$a\frac{d^2y_p}{dt^2} + b\frac{dy_p}{dt} + cy_p =$$

Let's see how this works in Example 1 from last class.

**Example 1.** Recall

$$\frac{d^2y}{dt^2} - \frac{dy}{dt} - 2y = e^{3t}.$$

Sinusoidal forcing

Now we are going to study forced equations where the forcing function is sinusoidal (either sine or cosine).

**Example.** Let's calculate the general solution to the equation

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} + 2y = \cos 2t.$$

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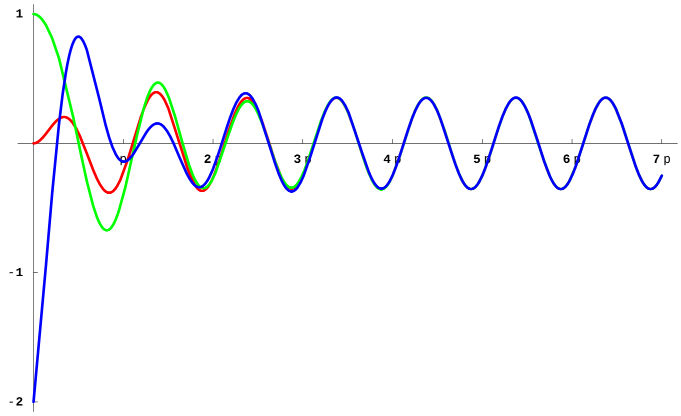
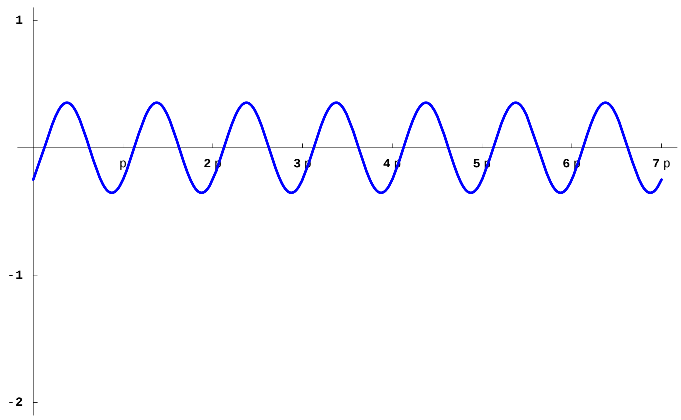
We can see the implications of this computation by entering this equation into `ForcedMassSpring` on the CD.

Here are the graphs of three solutions:




Here is the graph of the steady-state solution:




A little translation:

Consider the second-order linear forced equation

$$m\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ky = f(t)$$

where $m$ and $k$ are positive and $b \geq 0$.

Engineering terminology:

**forced response**—any solution to the forced equation.

**steady-state response**—behavior of the forced response over the long term.

**natural (or free) response**—any solution of the associated homogeneous equation.

Why are initial conditions essentially irrelevant?

When we guess a solution of the form $y_c(t) = ae^{i\omega t}$ and compute the complex number $a$, we have essentially determined everything we need to know about the steady-state solution. Euler's formula gives us a nice way of determining the amplitude, frequency, and phase angle of the steady-state solution immediately from $a$: