Separable Differential Equations (an analytic technique)

First let's recall the method of substitution for calculating integrals (really antiderivatives):

A differential equation

$$\frac{dy}{dt} = f(t, y)$$

is **separable** if it can be written in the form

$$\frac{dy}{dt} =$$

Two Examples:

1. $\dfrac{dy}{dt} = -2ty^2$

2. $\dfrac{dy}{dt} = y^3 + t^2$

Let's go back to the first example

Example. $\frac{dy}{dt} = -2ty^2$

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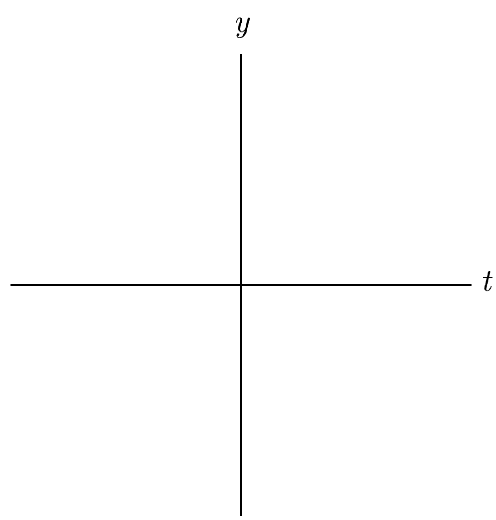
We turn to `FirstOrderExamples` to get a sense of the graphs of these solutions:

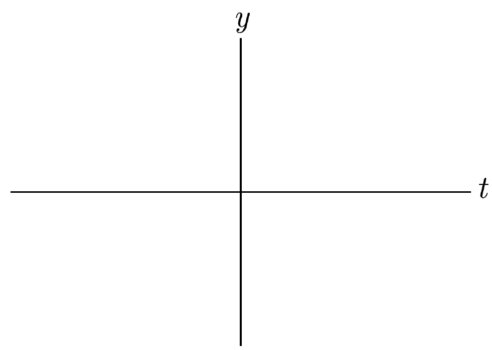


What's the general solution to $\dfrac{dy}{dt} = -2ty^2$? (Think before you answer.)

Slope fields:

A **slope field** in the ty-plane is the geometric manifestation of a first-order differential equation

$$\frac{dy}{dt} = f(t, y).$$

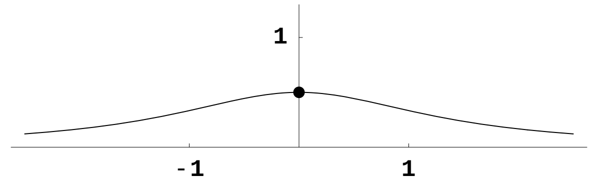


The graph of a solution must be everywhere tangent to the slope field.

Example. Consider the differential equation

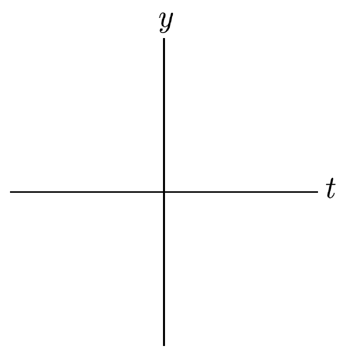
$$\frac{dy}{dt} = -2ty^2$$

on which we separated variables.

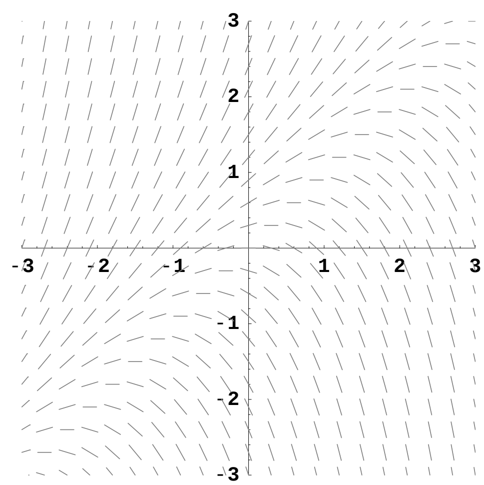


Example. Consider the differential equation

$$\frac{dy}{dt} = y - t.$$



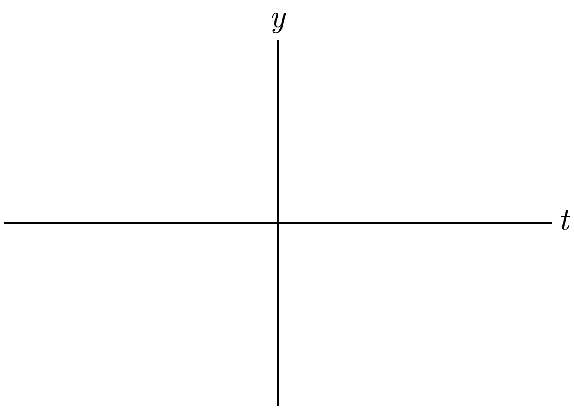
Using the computer to plot the slope field, we get

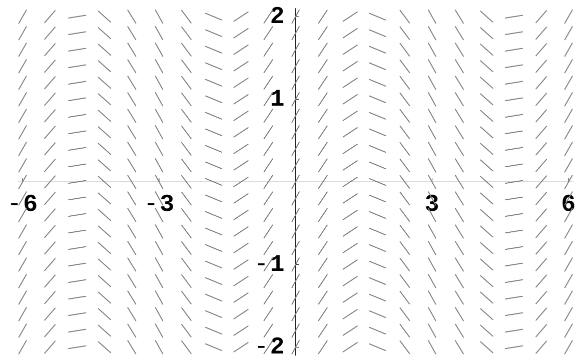


The `TargetPractice` tool is a fun way to see how the slope field relates to the solutions of the differential equation.

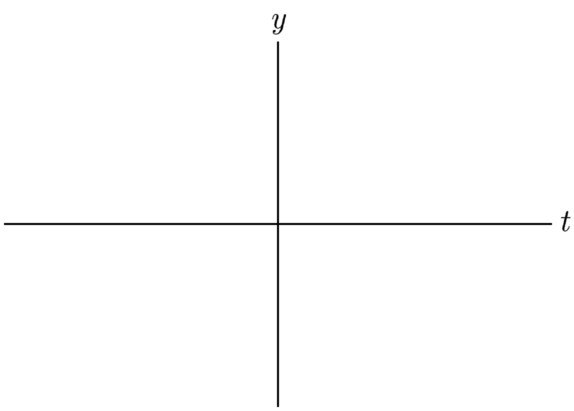
Important special cases:

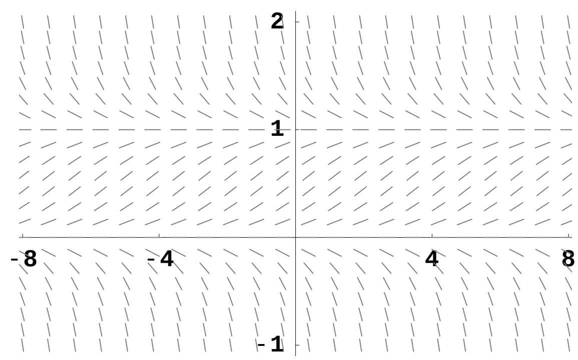
1. $\dfrac{dy}{dt} = f(t)$





2. $\dfrac{dy}{dt} = f(y)$ (autonomous)





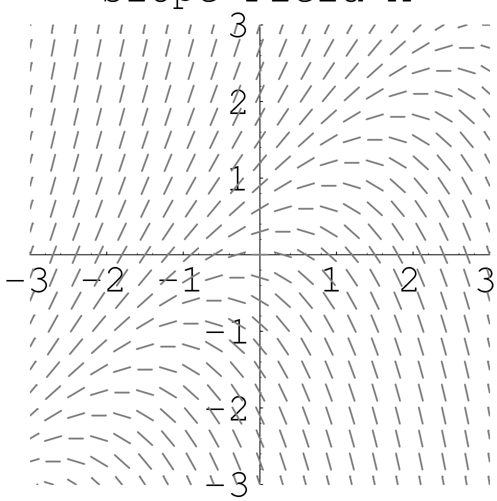
Typical exam problem:

Consider the following 8 first-order equations:

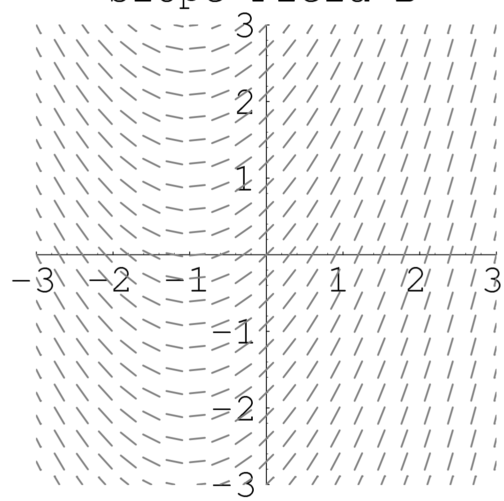
1. $\frac{dy}{dt} = t - 1$
2. $\frac{dy}{dt} = t + 1$
3. $\frac{dy}{dt} = y + 1$
4. $\frac{dy}{dt} = 1 - y$
5. $\frac{dy}{dt} = y^2 + y$
6. $\frac{dy}{dt} = y(y^2 - 1)$
7. $\frac{dy}{dt} = y - t$
8. $\frac{dy}{dt} = y + t$

Four of the associated slope fields are shown below. Pair the slope fields with their associated equations. Provide a brief justification for your choice.

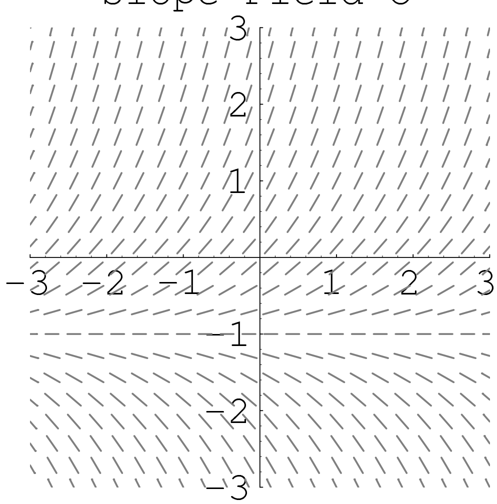
Slope Field A



Slope Field B



Slope Field C



Slope Field D