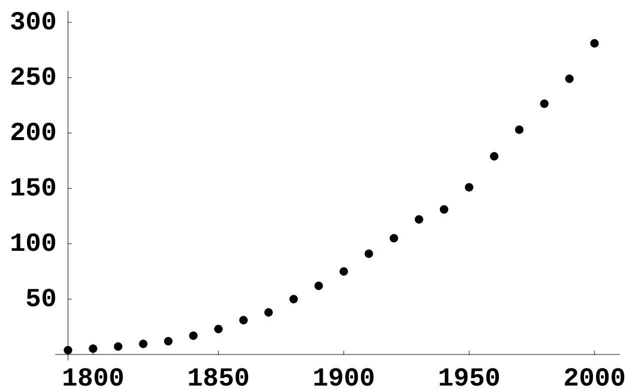
Modeling the US Population:

The data graphed as a function of time



Steps in Model Building

1. State underlying assumptions.
2. Identify the relevant variables and parameters.
3. Use the assumptions in Step #1 to formulate equations relating the variables in Step #2.

First Model: Malthusian Model

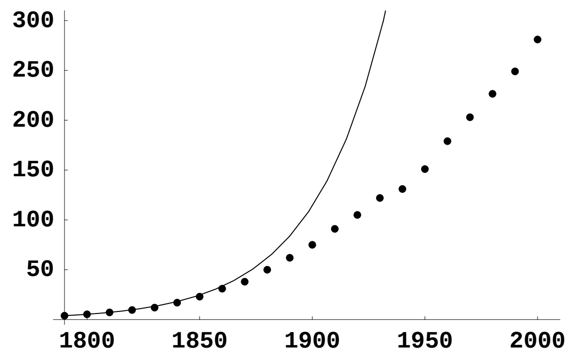
Assumption: Growth rate of the population is proportional to the population.

Variables:

Malthusian model is

Analytic technique:

Here's the graph of $p(t)$ superimposed on the data:



Second Model: Logistic Model
Assumptions:

1. If the population is small, its growth rate is proportional to the size of the population.
2. As the population increases, its **relative growth rate** decreases.

What is a relative growth rate?

A Qualitative Analysis of the Logistic Model

We now have

$$\frac{dp}{dt} = kp\left(1 - \frac{p}{N}\right).$$

Can we determine the long-term behavior of solutions without computing the solutions first?

A Numerical Simulation of the Logistic for the US Population

If we want to study this model numerically, we need estimates for k and N. How do we approximate the relative growth rates from the data?

Let's start by approximating the relative growth rate at 1800:

We can repeat this computation to produce approximate relative growth rates for 1800–1990:

Year	U.S. Population	Rel Growth Rate
1800	5.3	0.03113
1810	7.2	0.02986
1820	9.6	0.02500
1830	12	0.03083
1840	17	0.03235
1850	23	0.03043
1860	31	0.02419
1870	38	0.02500
1880	50	0.02400
1890	62	0.02016
1900	75	0.01933
1910	91	0.01648
1920	105	0.01476
1930	122	0.01066
1940	131	0.01107
1950	151	0.01589
1960	179	0.01453
1970	203	0.01170
1980	226	0.01015
1990	249	0.01094

Here's a graph of these relative growth rates versus population: