More on the diagonalization problem

Last class we learned that the only way that an $n \times n$ matrix $A$ can be diagonalized is if there is a basis of $\mathbb{R}^n$ of eigenvectors for $A$.

Let’s see why having such a basis is enough to be able to diagonalize $A$: Suppose $A$ has $n$ linearly independent eigenvectors

$$\{v_1, v_2, \ldots, v_n\}$$

and let $\lambda_i$ be the eigenvalue that is associated to $v_i$. (Note: The $\lambda_i$ need not be distinct.)

Then we can diagonalize $A$ using the matrix

$$P = \begin{bmatrix} v_1 & v_2 & \ldots & v_n \end{bmatrix}.$$
Example. Let

\[ A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}. \]
Now let’s return to the unusual matrix that is in the animation.

**Example.** Consider the matrix

\[
A = \begin{bmatrix}
31 & 19 \\
45 & 45 \\
19 & 119 \\
90 & 90
\end{bmatrix}.
\]
Example. Let

\[ A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 2 & -1 \\ 1 & 1 & 0 \end{bmatrix}. \]