MA 713
Reading and Exercises
Week ending January 21

Reading:

Class 1 (1/19): Ahlfors pp. 1–20

Class 2 (1/21): Ahlfors pp. 21–24

Exercises to be submitted for grading on Friday, January 28:

Class 1 (1/19):
Ahlfors Exercise 3 on p. 9
Ahlfors Exercise 1 on p. 11

Class 2 (1/21):
Ahlfors Exercise 3 on p. 16 (roots in algebraic form)

Additional Exercise 1: In class, we defined the map $\varphi_1(z) : \mathbb{C} \to S^2 \subset \mathbb{R}^3$ to be the inverse of stereographic projection. Define a similar map $\varphi_2(z) : \mathbb{C} \to S^2$ that omits the "south pole" and that satisfies

$$\varphi_1^{-1} \circ \varphi_2(z) = \frac{1}{z}$$

for all $z \neq 0$.

Additional Exercise 2: Prove that stereographic projection is a conformal map. (Recall that stereographic projection takes lines and circles in $\mathbb{C}$ to circles on $S^2$. To show that it is conformal, it is enough to show that it preserves the angles between any two intersecting lines in $\mathbb{C}$ when it maps those lines to circles on $S^2$. Either a geometric or an analytic proof is acceptable.)

Additional Exercise 3: We can think of $\varphi_2(z)$ in Additional Exercise 1 as defining another coordinate system, $w = 1/z$, on $\overline{\mathbb{C}} - \{0\}$. For the polynomial $p(z) = z^2 - 1$, find a formula $p(w)$ for this function in terms of this new coordinate system. Then use this formula to show that the derivative at "infinity" is zero.

Optional exercises:

Class 1 (1/19):
Ahlfors Exercises 1 and 2 on p. 6
Ahlfors Exercises 1 and 5 on p. 8
Ahlfors Exercise 3 on p. 11

Class 2 (1/21):
Ahlfors Exercise 2 on p. 15
Ahlfors Exercise 4 on p. 16
Ahlfors Exercises 1 and 5 on the bottom of p. 17