The following are problems similar to those that may appear on the first mid-term. So doing these problems is a good, but you should also review the homework problems, text, notes, etc..

JUSTIFY ALL ANSWERS. NEATNESS DEFINITELY COUNTS.

1. TRUE or FALSE and GIVE REASON: State if the following is true of fals and justify your answer in a sentence (or two at the most).

   (a) Suppose you have a sequence $a_1, a_2, \ldots$ that satisfies

   $$\lim_{n\to\infty} a_n = 5$$

   then

   $$\lim_{n\to\infty} a_{2n} = 10.$$

   (b) Suppose you have a sequence $a_1, a_2, \ldots$ that satisfies

   $$\sum_{n=0}^{\infty} a_n$$

   converges, then

   $$\sum_{n=0}^{\infty} a_{2n}$$

   also converges.

2. Compute

   $$\lim_{n\to\infty} \frac{n^2 + 5n + 7}{3n^2 - 4n + \ln(n)}$$

3. Determine if the following converge or diverge. If it converges state the value if you can.

   (a)

   $$\sum_{n=0}^{\infty} \left(\frac{4}{5}\right)^n$$

   (b)

   $$\sum_{n=1}^{\infty} (0.3)^{2n}$$

   (c)

   $$\sum_{n=1}^{\infty} \left(\frac{n+3}{\sqrt{n^6 + 3n + 2}}\right)$$

4. Use the idea behind the integral test to give an estimate on the size of

   $$\sum_{n=1}^{100} \frac{n^2}{n^3 + 3}$$

   (Be sure to include a figure which helps to explain what you are doing AND be neat!)

5. What is the radius of convergence of

(a)

$$\sum_{n=0}^{\infty} 3nx^n$$

(b)

$$\sum_{n=0}^{\infty} \frac{(x-2)^n}{3n!}$$

6. (a) What it the 5th degree Taylor polynomial for $\sin(x)$ centered at 0?
   (b) What is the maximum error between the 5th degree Taylor polynomial for $\sin(x)$ centered at 0 and $\sin(x)$ on the interval $-2 < x < 2$?
   (c) What is the maximum error between the 5th degree Taylor polynomial for $\sin(x)$ centered at 0 and $\sin(x)$ on the interval $-1 < x < 1$?
7. What is the Taylor polynomial of 3rd degree of $\ln(x)$ centered at $x = 2$.
8. The gravitational acceleration on a body near the earth is given by

$$A(x) = -\frac{GM}{x^2}$$

   where G is the gravitational constant, M is the mass of the earth and x is the distance to the center of the earth. Let r_0 be the radius of the earth.

   (a) Give the Taylor series expansion of degree 3 for $A(x)$ centered at $x = r_0$. (So your answer will have constants G, M and r_0).
   (b) How close is the 2nd degree Taylor polynomial for $A(x)$ centered at r_0 to the value of $A(x)$ on the interval $r_0 \leq x \leq 2r_0$?